Allowable Stress Design

Load Combinations

(1) D
(2) D + L
(3) D + (L_r or S or R)
(4) D + 0.75L + 0.75(L_r or S or R)
(5) D + 0.6W
(6) D + 0.75L + 0.75(0.6W) + 0.75(L_r or S or R)
(7) 0.6D + 0.6W
(8) 1.0D + 0.7E_v + 0.7E_h
(9) 1.0D + 0.525E_v + 0.525E_h + 0.75L + 0.75S
(10) 0.6D - 0.7E_v + 0.7E_h

It shall be permitted to replace 0.6D with 0.9D in combination (10) for the design of Special Reinforced Masonry Shear Walls

Assumptions:
1. Plane sections remain plane
2. Stress-strain relationship for masonry is linear in compression
3. All masonry in tension is neglected
4. Perfect bond between steel and grout
5. Member is straight prismatic section

Notation:
Lower case: calculated stress, $f_s$
Upper case: allowable stress, $F_s$
$F_h$ is allowable compressive stress to resist flexure only. Notes use $F_m$ for allowable compressive stress to resist combinations of flexure and axial load.
Flexural Members

To find neutral axis, equate moments of areas about neutral axis.

\[(bkd) \left( \frac{1}{2} kd \right) = (n\rho bd)(d - kd)\]

\[nA_s = npbd\]

Transformed Section

Steel stress: \(f_s = \frac{M}{A_s j d}\)

Masonry stress: \(f_m = \frac{2M}{b(kd)(jd)}\)

Allowable moment:

Steel: \(M_s = A_s F_s j d\)  
Masonry: \(M_m = b(kd) \frac{f_m}{2} (jd)\)

Example: Masonry Beam

Given: \(M = 340 \text{k-in}; \) Grade 60 steel, \(f_m' = 2000 \text{psi}; \) 8 in CMU; Type S mortar; 4 course high beam \((d = 28 \text{ in.}); \) #6 rebar

Required: Is section adequate?

Solution:

\[F_m = \]
\[F_s =\]
\[E_m = 900f_m' = 1.80 \times 10^6 \text{psi}\]
\[E_s = 20 \times 10^6 \text{psi}\]
\[n = E_s/E_m = \]
\[\rho = A_s/bd =\]
\[np = 16.1(0.00206) = 0.0332\]

\[k = \sqrt{(np)^2 + 2np} - np = \sqrt{(0.0332)^2 + 2(0.0332)} - 0.0332 =\]

\[j = 1 - \frac{k}{3} = 1 - \frac{0.227}{3} =\]
Example: Masonry Beam

Given:  \( M = 340 \text{k-in} \); Grade 60 steel, \( f_m' = 2000 \text{psi} \); 8 in CMU; Type S mortar; 4 course high beam \((d = 28 \text{ in.})\); #6 rebar

Required: Is section adequate?

Solution:

- \( F_m = 0.45(2000 \text{psi}) = 900 \text{psi} \)
- \( F_s = 32000 \text{psi} \)
- \( E_m = 900f_m' = 1.80 \times 10^6 \text{psi} \)
- \( E_s = 20 \times 10^6 \text{psi} \)
- \( n = E_s/E_m = 16.1 \)
- \( \rho = A_s/bd = 0.44 \text{in.}^2/(7.625\text{in.} \times 28 \text{in.}) = 0.00206 \)
- \( n\rho = 16.1(0.00206) = 0.0332 \)

\[
k = \sqrt{(n\rho)^2 + 2n\rho - n\rho} = \sqrt{(0.0332)^2 + 2(0.0332) - 0.0332} = 0.227
\]

\[
j = 1 - \frac{k}{3} = 1 - \frac{0.227}{3} = 0.924
\]

What is maximum moment beam could carry?

- \( M_s = A_sF_s/d = (0.44 \text{in.}^2)(32 \text{ksi})(0.924)(28 \text{in.}) = 364 \text{k \cdot in.} \)

- \( M_m = b(kd)\frac{F_m}{2}\frac{900 \text{psi}}{2}(0.924)(28 \text{in.})^2 = 564 \text{k \cdot in.} \)

\( M_{\text{all}} = 364 \text{ kip-in} \)
Example: Masonry Beam

\[ f_s = \frac{M}{A_s d} = \frac{340 \text{ kips} \cdot \text{in.}}{(0.44 \text{ in.}^2)(0.924)(28 \text{ in.})} = 29.8 \text{ksi} \leq 32 \text{ksi} \quad \text{OK} \]

\[ f_m = \frac{2M}{b(kd)(jd)} = \frac{2(340 \text{ kips} \cdot \text{in.})}{7.625 \text{ in.}(0.227)(28 \text{ in.})(0.924)(28 \text{ in.})} = 543 \text{psi} \leq 900 \text{psi} \quad \text{OK} \]

Beam is good

What is maximum moment beam could carry?

\[ M_s = A_s F_s d = (0.44 \text{ in.}^2)(32 \text{ksi})(0.924)(28 \text{ in.}) = 364 \text{kips} \cdot \text{in.} \]

\[ M_m = b(kd) \frac{F_m}{2}(jd) = 7.625 \text{ in.}(0.227) \frac{900 \text{psi}}{2}(0.924)(28 \text{ in.})^2 = 564 \text{kips} \cdot \text{in.} \]

\[ M_{all} = 364 \text{ kip-in} \]

Masonry Beam - Parametric Study

![Graph showing the relationship between allowable moment (M_allow) and area of steel (A_s) for different masonry strengths (f_m)].

- \( f_m = 3200 \text{ psi} \)
- \( f_m = 2800 \text{ psi} \)
- \( f_m = 2400 \text{ psi} \)
- \( f_m = 2000 \text{ psi} \)

- \( d = 20 \text{ inch} \)
- \( b = 7.625 \text{ inch} \)
Design Procedure

1. Assume value of $j$ (or $k$). Typically $0.85 < j < 0.95$.
2. Determine a trial value of $A_{s,reqd}$.
   $$A_{s,reqd} = \frac{M}{(F_s j d)}$$
   Choose reinforcement.
3. Determine $k$ and $j$; steel stress and masonry stress.
4. Compare calculated stresses to allowable stresses.
5. If masonry stress controls design, consider other options (such as change of member size, or change of $f'_{m}$). Reinforcement is not being used efficiently.
### Design Procedure

#### Calculate

\[ kd = 3 \left[ d - \sqrt{\left(\frac{d}{2}\right)^2 - \frac{2M}{3F_b t_{sp}}} \right] \]

**Is \( k \geq k_{bal} \)?**

For Grade 60 steel, CMU \( k_{bal} = 0.312 \)

**YES**

\[ A_{s,reqd} = \frac{F_b (kd) t_{sp}}{2nF_b \left( \frac{1}{k} - 1 \right)} \]

Compression controls

**NO**

\[ A_{s,reqd} = \frac{M}{F_s d \left( 1 - k \right)} \]

\[ \zeta = \frac{A_{s,reqd} F_s n}{F_s t_{sp}} \]

\[ (kd)_2 = \sqrt{\zeta^2 + 2\zeta d - \zeta} \]

Tension controls

Iterate. Use \((kd)_2\) as new guess and repeat.

---

### Example: Beam Design

**Given:** 10 ft. opening; dead load of 1.5 kip/ft; live load of 1.5 kip/ft; 24 in. high; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; \( f'_m = 2000 \) psi

**Required:** Design beam

**Solution:**

1. **5.2.1.3:** Length of bearing of beams shall be a minimum of 4 in.; **typically assumed to be 8 in.**

2. **5.2.1.1.1** Span length of members not built integrally with supports shall be taken as the clear span plus depth of member, but need not exceed distance between center of supports.
   - Span = 10 ft + 2(4 in.) = 10.67 ft

3. **5.2.1.2** Compression face of beams shall be laterally supported at a maximum spacing of:
   - 32 multiplied by the beam thickness. 32(7.625 in.) = 244 in. = 20.3 ft
   - 120\( b^2/d \). 120(7.625 in.)^2 / (20 in.) = 349 in. = 29.1 ft
Example: Beam Design

Load
Weight of fully grouted normal weight: 83 psf

\[ w = D + L = \left( 1.5 \frac{k}{ft} + 0.083 \frac{k}{ft^2} (2ft) \right) + 1.5 \frac{k}{ft} = 3.17 \frac{k}{ft} \]

Moment

\[ M = \frac{wL^2}{8} = \frac{(3.17\frac{k}{ft})(10.67ft)^2}{8} = 45.1k \cdot ft \]

Determine \( kd \) Assume compression controls

\[ kd = 3 \left[ \frac{d}{2} - \sqrt{\left( \frac{d}{2} \right)^2 - \frac{2M}{3F_b b}} \right] = 3 \left[ \frac{20in.}{2} - \sqrt{\left( \frac{20in.}{2} \right)^2 - \frac{2(45.2k-ft)(12in.)}{3(0.90ksi)(7.625in.)}} \right] = 9.32in. \]

Check if compression controls

\[ k = \frac{kd}{d} = \frac{9.32in.}{20in.} = 0.466 > 0.312 \text{ Compression controls} \]

Calculate modular ratio, \( n \)

\[ n = \frac{E_s}{E_m} = \frac{E_s}{900f_m^t} = \frac{29000ksi}{900(2.0ksi)} = 16.1 \]

Example: Beam Design

Find \( A_{s,reqd} \)

Req'd area of steel

\[ A_{s,reqd} = \frac{F_b (kd)b}{nF_b \left( \frac{1}{k} - 1 \right)} = \frac{0.90ksi(9.31in.)7.625in.}{16.1(0.90ksi) \left( \frac{1}{0.466} - 1 \right)} = 1.94in.^2 \]

Use 2 - #9 (\( A_s = 2.00 \text{ in}^2 \))

Now that is some reinforcement that would make me proud!
Example: ASD vs. SD

<table>
<thead>
<tr>
<th>Dead Load (k/ft) (superimposed)</th>
<th>Live Load (k/ft)</th>
<th>Required $A_s$ (in²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>ASD 0.34, SD 0.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.34 (f'_m = 1.5 \text{ ksi})$, $0.26 (f'_m = 1.5 \text{ ksi})$</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>ASD 0.64, SD 0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.65 (f'_m = 1.5 \text{ ksi})$, $0.52 (f'_m = 1.5 \text{ ksi})$</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5</td>
<td>ASD 1.94, SD 0.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$5.09 (f'_m = 1.5 \text{ ksi})$, $0.80 (f'_m = 1.5 \text{ ksi})$</td>
</tr>
</tbody>
</table>

ASD: Allowable tension controls for 0.5 k/ft and 1 k/ft.

Partially Grouted Walls

A. Neutral axis in flange; design and analysis for solid section
B. Neutral axis in web

\[
k = \frac{b}{b'} \sqrt{\frac{t_f}{a^2} \left( 1 - \frac{b'}{b} \right) + (\rho n)^2 + 2\rho n \left( \frac{t_f}{d} + \frac{b'}{b} - \frac{b'b'}{b'd} \right) - \rho n \frac{b}{b'} + \left( 1 - \frac{b}{b'} \right) \frac{t_f}{d}}
\]

\[
C_f = \frac{f_m}{2} \left( \frac{2kd - t_f}{kd} \right) b t_f
\]

\[
C_w = \frac{f_m}{2} \left( \frac{kd - t_f}{kd} \right) b'(kd - t_f)
\]

\[
j_f = 1 - \frac{t_f}{3d} \left( \frac{3kd - 2t_f}{2kd - t_f} \right)
\]

\[
j_w = 1 - \frac{2t_f + kd}{3d}
\]

\[
M = C_f j_f d + C_w j_w d
\]

\[
f_m = F_m \text{ if masonry controlling}
\]

\[
f_m = F_s k / (n(1 - k)) \text{ if steel controlling}
\]
Example: Partially Grouted Walls

Given: 8 in CMU wall; 16 ft high; Grade 60 steel, \( f_m' = 2000 \text{ psi} \); Lateral wind load of \( w_u = 30 \text{ psf} \) (factored)

Required: Reinforcing (place in center of wall)

Solution:

\[ M = \frac{wh^2}{8} = \frac{0.6(30 \text{ lb/ft})(12 \text{ in./ft})(16 \text{ ft})^2}{8} = 6912 \text{ lb-in./ft} = 576 \text{ lb-ft/ft} \]

\[ F_b = 0.45(2000 \text{ psi}) = 900 \text{ psi} \]

\[ F_s = 32000 \text{ psi} \]

\[ n = \frac{E_s}{E_m} = 16.1 \]

Determine \( kd \) Assume compression controls

\[ kd = 3 \left[ \frac{d^2}{2} - \sqrt{\left( \frac{d}{2} \right)^2 - \frac{2M}{3F_b}} \right] = 3 \left[ \frac{3.81 \text{ in.}^2}{2} - \sqrt{\left( \frac{3.81 \text{ in.}}{2} \right)^2 - \frac{2(0.576 \text{ ft})}{3(0.90 \text{ ksi})}} \right] = 0.346 \text{ in.} \]

Check if compression controls

\[ k = \frac{kd}{d} = \frac{0.346 \text{ in.}}{3.81 \text{ in.}} = 0.091 < 0.312 \]

Tension controls

---

<table>
<thead>
<tr>
<th>( kd ) (in.)</th>
<th>Iteration 1</th>
<th>Iteration 2</th>
<th>Iteration 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.346</td>
<td>0.699</td>
<td>0.709</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( k )</th>
<th>Iteration 1</th>
<th>Iteration 2</th>
<th>Iteration 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.091</td>
<td>0.183</td>
<td>0.183</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( A_{s,reqd} = \frac{M}{F_s d (1 - \frac{k^2}{3})} ) (in.²)</th>
<th>Iteration 1</th>
<th>Iteration 2</th>
<th>Iteration 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0584</td>
<td>0.0603</td>
<td>0.0603</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \zeta = \frac{A_{s,reqd}F_s}{F_s \zeta sp} ) (in.)</th>
<th>Iteration 1</th>
<th>Iteration 2</th>
<th>Iteration 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0784</td>
<td>0.0810</td>
<td>0.0810</td>
<td></td>
</tr>
</tbody>
</table>

\[ (kd)_2 = \sqrt{\zeta^2 + 2\zeta d} - \zeta \] (in.)

0.699          0.709          0.709

Use # 4 @ 40 inches \((A_s = 0.060 \text{ in}^2/\text{ft})\)

(close enough)
**Axial Strength**

Allowable Compressive Force (8.3.4.2.1)

\[ P_a = (0.25f_m' A_n + 0.65A_{st} F_s) \left[ 1 - \left( \frac{h}{140r} \right)^2 \right] \text{ for } \frac{h}{r} \leq 99 \]

\[ P_a = (0.25f_m' A_n + 0.65A_{st} F_s) \left( \frac{70r}{h} \right)^2 \text{ for } \frac{h}{r} > 99 \]

\( A_{st} \) is area of laterally tied steel

---

**Interaction Diagram**

- Assume strain/stress distribution
  - For \( k > k_{bal} \) Set masonry strain, find steel strain
    - Masonry strain = \( F_m/E_m \) = 0.0005 for CMU
  - For \( k \leq k_{bal} \) Set steel strain, find masonry strain
    - Steel strain = \( F_s/E_s \) = 0.00110 for Grade 60
- Compute forces in masonry and steel
- Sum forces; sum moments about centerline

Grade 60 steel

\[ k_{bal} = \frac{F_m}{F_m + F_s/n} = \frac{F_m}{E_m + F_s/E_s} = \frac{0.45f_v'}{900f_m'} + \frac{32kst}{29000kst} = 0.312 \]
Example: Interaction Diagram

**Given:** 12 ft high CMU bearing wall, Type S masonry cement mortar; Grade 60 steel in center of wall; #4 @ 48 in.; partial grout; \( f_m' = 2000 \text{ psi} \)

**Required:** Interaction diagram in terms of capacity per foot

**Pure Moment:** \( n = 16.1 \quad \rho = 0.00109 \quad n\rho = 0.0176 \)

Depth to NA \( k \)

\[
k = \sqrt{(n\rho)^2 + 2n\rho - n\rho} = \sqrt{(0.0176)^2 + 2(0.0176) - 0.0176} = 0.171
\]

Internal lever arm \( j \)

\[
j = 1 - \frac{k}{3} = 1 - \frac{0.171}{3} = 0.943
\]

Steel moment \( M_s \)

\[
M_s = A_s F_s j d = \left(0.05\frac{\text{in}^2}{\text{ft}}\right)(32\text{ksi})(0.943)(3.81\text{in.}) = 5.75\frac{k\cdot\text{in.}}{\text{ft}}
\]

Masonry moment \( M_m \)

\[
M_m = b(kd)\frac{f_m'}{2}(jd) = 12\frac{\text{in}}{\text{ft}}\left(0.171\right)^{\frac{900\text{psi}}{2}}(0.943)(3.81\text{in.})^2
\]

\[
= 12.65\frac{k\cdot\text{in.}}{\text{ft}}
\]

**Allowable** \( M \)

\[
M = 5.75\frac{k\cdot\text{in.}}{\text{ft}} = 0.479\frac{k\cdot\text{ft}}{\text{ft}}
\]

Example: Interaction Diagram

**Pure Axial:**

\[4.3.2\text{ Radius of gyration} \]

Radius of gyration shall be computed using average net cross-sectional area of the member considered.

NCMA TEK 14-1B  Section Properties of Concrete Masonry Walls

\( r = 2.66 \text{ in.} \quad A_n = 40.7\text{in.}^2/\text{ft} \)

Slenderness \( h/r \)

\[
h \quad \frac{144\text{in.}}{2.66\text{in.}} = 54.1 \quad \leq 99
\]

Axial load \( P_a \)

\[
P_a = \left(0.25f_m'A_n + 0.65A_s F_s\right) \left[1 - \left(\frac{h}{140r}\right)^2\right]
\]

\[
P_a = \left(0.25(2.0\text{ksi})\left(40.7\frac{\text{in.}^2}{\text{ft}} + 0\right) \left[1 - \left(\frac{54.1}{140}\right)^2\right\right]
\]

\[
= 17.2\frac{k}{\text{ft}}
\]

**Allowable** \( P \)

\[
P = 17.2\frac{k}{\text{ft}}
\]
Example: Interaction Diagram

Balanced:

Masonry force \( C_m \)

Tension force \( T \)

Axial force \( P \)

Moment \( M \)

\[
kd = \frac{0.0005}{0.0005 + 0.00110} = 3.81 \text{ in.} = 1.19 \text{ in.}
\]

\( kd < 1.25 \text{ in.} \)

N.A. in face shell

\[
P = 4.83 \frac{k}{ft} \quad M = 1.83 \frac{k \cdot ft}{ft}
\]
**Example: Interaction Diagram**

**Below Balanced:**

\[ kd = 1.00 \text{ in.} \]

\[ \varepsilon_m = \frac{1.00 \text{ in.}}{3.81 \text{ in} - 1.00 \text{ in.}} \cdot 0.00110 = 0.000391 \]

\[ f_m = E_m \varepsilon_m = 1800 \text{ksi} (0.000391) = 0.704 \text{ksi} \]

Masonry force \( C_m \):

\[ C_m = \frac{1}{2} f_m (kd)b = \frac{1}{2} (0.704 \text{ksi})(1.00 \text{in.}) \left( 12 \frac{\text{in.}}{\text{ft}} \right) = 4.22 \frac{k}{\text{ft}} \]

Tension force \( T \):

\[ T = f_s A_s = (32 \text{ksi}) \left( 0.05 \frac{\text{in.}^2}{\text{ft}} \right) = 1.6 \frac{k}{\text{ft}} \]

Axial force \( P \):

\[ P = C_m - T = (4.22 - 1.60) \frac{k}{\text{ft}} = 2.62 \frac{k}{\text{ft}} \]

Moment \( M \):

\[ M = 4.22 \frac{k}{\text{ft}} \left( 3.81 \text{in.} - \frac{1.00 \text{in.}}{3} \right) = 14.7 \frac{k \cdot \text{in.}}{\text{ft}} = 1.22 \frac{k \cdot \text{ft}}{\text{ft}} \]

\[ P = 2.62 \frac{k}{\text{ft}} \quad M = 1.22 \frac{k \cdot \text{ft}}{\text{ft}} \]

Allowable Stress Design

---

**Example: Interaction Diagram**

**Above Balanced:**

\[ kd = 2.00 \text{ in.} \]

\[ \varepsilon_s = \frac{3.81 \text{in.} - 2.00 \text{in.}}{2.00 \text{in.}} \cdot 0.0005 = 0.00045 \]

\[ f_s = E_s \varepsilon_s = 29000 \text{ksi} (0.00045) = 13.1 \text{ksi} \]

Face shell force \( C_m \):

\[ C_m = \frac{1}{2} (0.900 + 0.338) \text{ksi} (1.25 \text{in.}) \left( 12 \frac{\text{in.}}{\text{ft}} \right) = 9.28 \frac{k}{\text{ft}} \]

Web force \( C_m \):

\[ C_m = \frac{1}{2} (0.338 \text{ksi})(2.00 \text{in.} - 1.25 \text{in.}) \left( 2 \frac{\text{in.}}{\text{ft}} \right) = 0.25 \frac{k}{\text{ft}} \]

Tension force \( T \):

\[ T = f_s A_s = (13.1 \text{ksi}) \left( 0.05 \frac{\text{in.}^2}{\text{ft}} \right) = 0.66 \frac{k}{\text{ft}} \]

Allowable Stress Design
**Example: Interaction Diagram**

Axial force $P$ 

$$ P = C_{m1} + C_{m2} - T = (9.28 + 0.25 - 0.66) \frac{k}{ft} = 8.87 \frac{k}{ft} $$

Moment $M$ 

$$ M = 9.28 \frac{k}{ft} (3.81\text{ in.} - 0.53\text{ in.}) + 0.25 \frac{k}{ft} (3.81\text{ in.} - 1.50\text{ in.}) $$

$$ M = 31.0 \frac{k\cdot \text{in}}{ft} = 2.58 \frac{k\cdot \text{ft}}{ft} $$

$P = 8.87 \frac{k}{ft}$  

$M = 2.58 \frac{k\cdot \text{ft}}{ft}$
**Approximate Interaction Diagram**

Three Point Approximation

- Zero axial load; moment capacity
- \( kd \) = flange thickness
- Zero moment; axial capacity

**Design Procedure**

\[
kd = 3 \left[ d - \frac{\left( \frac{d}{2} \right)^2}{\sqrt{\left( \frac{d}{2} \right)^2 + 2(P(d - d_v/2) + M) / 3F_m t_{sp}}} \right]
\]

Is \( k \geq k_{bal} \)?

- For Grade 60 steel, CMU \( k_{bal} = 0.312 \)

Is \( k \geq k_{bal} \)?

\[ k_{bal} = \frac{F_m}{F_m + F_s/n} \]

**YES**

\[
A_{s,reqd} = \frac{F_m(kd)t_{sp}}{2} - P
\]

Compression controls

**NO**

\[
M' = P \left( \frac{d_v}{2} - \frac{kd}{3} \right)
\]

\[
A_{s,reqd} = \frac{M - M'}{F_s d \left( 1 - \frac{k}{3} \right)}
\]

\[
\zeta = \frac{(P + A_{s,reqd}F_s)n}{F_s t_{sp}}
\]

\[
(kd)^2 = \sqrt{\zeta^2 + 2\zeta d - \zeta}
\]

Iterate. Use \((kd)^2\) as new guess and repeat.

Tension controls
Derivation of Design Equations

Sum forces: \[
\frac{1}{2} (kd) t_{sp} f_m - A_s f_s = P
\]

Sum moments: \[
\frac{1}{2} (kd) t_{sp} f_m \left( \frac{d_y}{2} - \frac{kd}{3} \right) + A_s f_s \left( d - \frac{d_y}{2} \right) = M
\]

If the masonry stress controls, set \(f_m = F_m\), solve for \(A_s f_s\) from sum of forces, and substitute for \(A_s f_s\) in moment equation.

\[
\frac{1}{2} (kd) t_{sp} f_m \left( \frac{d_y}{2} - \frac{kd}{3} \right) + A_s f_s \left( d - \frac{d_y}{2} \right) (d - \frac{d_y}{2}) = M
\]

This is a quadratic equation in \(kd\), which can be solved to obtain:

\[
kd = 3 \left[ \frac{d}{3} - \sqrt{\left( \frac{3}{d} \right)^2 - \frac{2(P(d-d_{w}/2)+M)}{3F_m t_{sp}}} \right]
\]

Solve for \(A_{s,reqd}\)

\[
A_{s,reqd} = \frac{F_m (kd) t_{sp} - P}{2 \sqrt{F_m \left( \frac{1}{k} - 1 \right)}}
\]

Derivation of Design Equations

If the steel stress controls, set \(f_s = F_s\), and find \(f_m\) in terms of \(F_s\).

\[
f_m = E_m \varepsilon_m = E_m \frac{kd}{d-kd} \varepsilon_s = E_m \frac{kd}{d-kd} E_s = \frac{F_s}{n} \frac{kd}{d-kd}
\]

Substitute into sum of forces, and solve for \(A_s f_s\).

\[
A_s f_s = \frac{1}{2} (kd) t_{sp} f_m - P = \frac{1}{2} (kd) t_{sp} \left( \frac{F_s}{n} \frac{kd}{d-kd} - P \right) (d - \frac{d_y}{2}) = P
\]

Now substitute into sum of moments

\[
\frac{1}{2} (kd) t_{sp} f_m \left( \frac{d_y}{2} - \frac{kd}{3} \right) + \frac{1}{2} (kd) t_{sp} \left( \frac{F_s}{n} \frac{kd}{d-kd} - P \right) \left( d - \frac{d_y}{2} \right) = M
\]

This is a cubic equation in \(kd\). Although there are analytical ways to solve a cubic equation, numerical solutions are usually the easiest.

\[
\frac{t_{sp} F_s}{6n} [kd]^3 - \frac{t_{sp} d F_s}{2n} [kd]^2 - \left( P \left( d - \frac{d_y}{2} \right) + M \right) [kd] + \left( P \left( d - \frac{d_y}{2} \right) + M \right) d = 0
\]

Solve for \(A_{s,reqd}\)

\[
A_{s,reqd} = \frac{1}{2} (kd) t_{sp} \left( \frac{1}{n} \frac{kd}{d-kd} \right) \frac{P}{F_s}
\]
Example: Pilaster

Given: Nominal 16 in. wide x 16 in. deep CMU pilaster; $f'_m = 2000$ psi; Grade 60 bar in each corner, center of cell; Effective height = 24 ft; Dead load of 9.6 kips and snow load of 9.6 kips act at an eccentricity of 5.8 in. (2 in. inside of face); Factored wind load of 26 psf (pressure and suction) and uplift of 8.1 kips ($e = 5.8$ in.); Pilasters spaced at 16 ft on center; Wall is assumed to span horizontally between pilasters; No ties.

Required: Determine required reinforcing using allowable stress design.

Solution:

Vertical Spanning

- $e = 5.8$ in
- $d = 11.8$ in
- $E_m = 1800$ ksi
- $n = 16.1$

Lateral Load
- $w = 0.6(26$ psf$)(16$ ft$) = 250$ lb/ft

$P_f = 0.6(9.6k) - 0.6(8.1k) = 0.9k$
$M_f = 0.9k(5.8$ in$.) = 5.2k \cdot$ in

Location of maximum moment

$$x = \frac{h}{2} - \frac{M_f}{wh} = \frac{288$ in$}{2} - \frac{5.2$ k-in$}{0.250$ k-ft$(24$ ft$)} = 143.1$ in.

Maximum moment

$$M_{max} = \frac{M_f}{2} + \frac{wh^2}{8} + \frac{M_f^2}{2wh^2}$$
$$= \frac{5.2$ k-in$}{2} + \frac{0.0208$ k-in$}{8} + \frac{(5.2$ k-in$)^2}{2(0.0208$ k-in$)(288$ in$)^2} = 218$ k-in.

Find axial force at this point. Include weight of pilaster (200 lb/ft).

$$P = 0.9k + 0.6\left(0.20 \frac{k}{ft}\right)(143.1$ in$.) \frac{1$ ft$}{12$ in$} = 2.3k$$

Design for $P = 2.3k$, $M = 218$ k-in.
**Example: Pilaster**

Assume compression controls; Determine $kd$

$$kd = 3 \left[ \frac{d}{2} - \sqrt{\left( \frac{d}{2} \right)^2 - \frac{2(P(d_d+L))/3F_{tsp}}{2}} \right]$$

$$= 3 \left[ \frac{11.8\text{in.}}{2} - \sqrt{\left( \frac{11.8\text{in.}}{2} \right)^2 - \frac{2(2.3k(11.8\text{in.} - 15.6\text{in.}/2) + 218k\text{in.})\left(\frac{12\text{in.}}{\text{ft}}\right)}{3(0.90\text{ksi})(15.6\text{in.})}} \right] = 3.00\text{in.}$$

Determine $k$

$$k = \frac{kd}{d} = \frac{3.00\text{in.}}{11.81\text{in.}} = 0.254 < 0.312$$

**Tension controls**

---

**Example: Pilaster**

<table>
<thead>
<tr>
<th>Equation / Value</th>
<th>Iteration 1</th>
<th>Iteration 2</th>
<th>Iteration 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$kd$ (in.)</td>
<td>3.00</td>
<td>3.38</td>
<td>3.40</td>
</tr>
<tr>
<td>$k$</td>
<td>0.254</td>
<td>0.286</td>
<td>0.288</td>
</tr>
<tr>
<td>$M' = P \left( \frac{d_d - kd}{3} \right)$ (k-in.)</td>
<td>15.6</td>
<td>15.3</td>
<td>15.3</td>
</tr>
<tr>
<td>$A_{s,reqd} = \frac{M - M'}{F_s d (1 - \frac{k}{3})}$ (in.$^2$)</td>
<td>0.585</td>
<td>0.593</td>
<td>0.593</td>
</tr>
<tr>
<td>$\zeta = \frac{(P + A_{s,reqd} F_s)}{F_s t_{sp}}$ (in.)</td>
<td>0.678</td>
<td>0.686</td>
<td>0.686</td>
</tr>
<tr>
<td>$(kd)_2 = \sqrt{\zeta^2 + 2\zeta d - \zeta}$ (in.)</td>
<td>3.38</td>
<td>3.40</td>
<td>3.40</td>
</tr>
</tbody>
</table>

Try 2 - #5, 4 total, one in each cell
**Example: Pilaster**

![Graph showing Allowable Values for different load combinations and their corresponding axial and moment values.]

- **Allowable Values**
  - $D + 0.75(0.6W) + 0.75S$
    - $P = 15.3k$
    - $M = 16.8k$-ft
  - $D + S$
    - $P = 19.2k$
    - $M = 16.8k$-ft
  - $0.6D + 0.6W$
    - $P = 7.1k$
    - $M = 19.2k$-ft
  - $0.6D + 0.6W$
    - $P = 2.3k$
    - $M = 18.2k$-ft

**Example: Effect of $f_m'$**

![Graph showing Allowable Values for different values of $f_m'$ and their corresponding axial and moment values.]

- **Allowable Values**
  - $f_m' = 2000$ psi
  - $f_m' = 1500$ psi

**Applied Loads**
- Red squares

**Balanced Point**
- Green squares
Example: ASD vs. Strength Design

![Graph showing Allowable Values]

Example: Bearing Wall

Given: 8 in. CMU wall; Grade 60 steel; Type S masonry cement mortar; \( f_m' = 2000 \text{ psi} \); roof forces act on 3 in. wide bearing plate at edge of wall.

Required: Reinforcement

Solution:

Estimate reinforcement

\[
M \sim \frac{wh^2}{8} = \frac{0.6(0.032\text{ksf})(18\text{ft})^2}{8} = 0.778 \text{k-ft/ft}
\]

Assume \( j = 0.95 \)

\[
A_{s,reqd} = \frac{M}{F_{s,j}d} = 0.080 \text{ in.}^2/\text{ft}
\]

Try #5 @ 48 in. (0.078 in.\(^2\)/ft)

Wall weight is 38 psf for 48 in. grout spacing

Cross-section of top of wall

Determine eccentricity

\( e = 7.625\text{in}/2 - 1.0 \text{ in.} = 2.81 \text{ in.} \)
Example: Bearing Wall

Check 0.6D+0.6W

Find force at top of wall
\[ P_f = 0.6 \left( \frac{0.5}{\text{kip/ft}} \right) + 0.6 \left( -\frac{0.36}{\text{kip/ft}} \right) = 0.084 \text{ kip/ft} \]

Find force at midheight
\[ P = 0.084 \text{ kip/ft} + 0.6(0.040\text{ksf})(2.67\text{ft} + 9\text{ft}) = 0.364 \text{ kip/ft} \]

Find moment at top of wall
\[ M_f = 0.084 \text{ kip/ft} \left( \frac{2.81}{12} \text{ ft} \right) - 0.6(0.032\text{ksf})(2.67\text{ft})^2 = -0.049 \frac{\text{k-ft}}{\text{ft}} \]

Find moment at midheight
\[ M = \frac{wh^2}{8} + \frac{M_f}{2} = \frac{0.6(0.032\text{ksf})(18\text{ft})^2}{8} + \frac{-0.049\frac{\text{k-ft}}{\text{ft}}}{2} = 0.753 \frac{\text{k-ft}}{\text{ft}} \]

<table>
<thead>
<tr>
<th>Load Comb.</th>
<th>( P_f ) (kip/ft)</th>
<th>( P ) (kip/ft)</th>
<th>( M_f ) (k-ft/ft)</th>
<th>( M ) (k-ft/ft)</th>
<th>( A_{s,reqd} ) (in.(^2/\text{ft}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6D+0.6W</td>
<td>0.084</td>
<td>0.364</td>
<td>-0.049</td>
<td>0.753</td>
<td>0.068</td>
</tr>
<tr>
<td>D+0.6W</td>
<td>0.284</td>
<td>0.751</td>
<td>-0.002</td>
<td>0.777</td>
<td>0.059</td>
</tr>
</tbody>
</table>

Use #5 @ 48 in. (0.078 in.\(^2/\text{ft}\))

Although close to #5 @ 56 in. (0.066in.\(^2/\text{ft}\)), a wider spacing also reduces wall weight

Moment is 90% of allowable moment;
In SD factored moment was 91% of design moment

Example: Bearing Wall

Sample Calculations: 0.6D+0.6W

1. \( k\text{bal} = 0.312; \ k\text{dibal} = 1.19\text{in.} \)
2. Assume masonry controls.
   Determine \( kd \).
   Since 0.478 in. < 1.18 in.
tension controls.
3. Iterate to find \( A_{s,reqd} \).

\[
k d = 3 \left[ \frac{d}{2} - \frac{\left( \frac{3.81\text{in.}}{2} \right)^2}{\left( \frac{3.81\text{in.}}{2} \right)^2 - 2\left( \frac{0.753\frac{\text{k-ft}}{\text{R}}} {3(0.90\text{ksi})}\right)\left( \frac{12\text{in.}}{\text{R}} \right)} \right] = 0.457\text{in.}
\]

For centered reinforcement, \((d - d_v/2) = 0\)

<table>
<thead>
<tr>
<th>Equation / Value</th>
<th>Iteration 1</th>
<th>Iteration 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( kd ) (in.)</td>
<td>0.457</td>
<td>0.791</td>
</tr>
<tr>
<td>( M' = P(d_v/2 - kd/3) ) (k-ft/ft)</td>
<td>0.1110</td>
<td>0.1076</td>
</tr>
<tr>
<td>( A_{s,reqd} = \frac{M' - M_t}{F_s d \left( 1 - \frac{k}{3} \right)} ) (in.(^2/\text{ft}))</td>
<td>0.0658</td>
<td>0.0682</td>
</tr>
<tr>
<td>( \zeta = \frac{(P + A_{s,reqd} F_s) n}{F_s t_{sp}} ) (in.)</td>
<td>0.1036</td>
<td></td>
</tr>
<tr>
<td>( (kd)^2 = \sqrt{\zeta^2 + 2\zeta d - \zeta} ) (in.)</td>
<td>0.791</td>
<td></td>
</tr>
</tbody>
</table>
Example: Bearing Wall

Allowable Shear Stress:

\[ F_v = (F_{vm} + F_{vs})\gamma_g \]

Allowable Masonry Stress:

\[ F_{vm} = \frac{1}{2} \left( 4.0 - 1.75 \left( \frac{M}{Vd_v} \right) \sqrt{f_m'} \right) + 0.25 \frac{P}{A_n} \]

Allowable Steel Stress:

\[ F_{vs} = 0.5 \left( \frac{A_v F_s d_v}{A_{nv} S} \right) \]

Maximum Shear Stress:

\[ F_v \leq \left( 3\sqrt{f_m'} \right)\gamma_g \quad (M/Vd_v) \leq 0.25 \]

\[ F_v \leq \left( 2\sqrt{f_m'} \right)\gamma_g \quad (M/Vd_v) \geq 1.0 \]

\[ F_v \leq \left( \frac{2}{3} \left( 5 - 2 \frac{M}{Vd_v} \right) \sqrt{f_m'} \right)\gamma_g \quad 0.25 < (M/Vd_v) < 1.0 \]

Special Reinforced Shear Walls

\[ F_{vm} = \frac{1}{4} \left( 4.0 - 1.75 \left( \frac{M}{Vd_v} \right) \sqrt{f_m'} \right) + 0.25 \frac{P}{A_n} \]

Masonry allowable shear stress decreased by a factor of 2, from ½ to ¼.

Seismic design load required to be increased by 1.5 for shear

Maximum reinforcement: Shear walls having

- \[ M/(Vd_v) \geq 1 \] and
- \[ P > 0.05f_m' A_n \]

\[ \rho_{max} = \frac{n f_m'}{2 f_y \left( n + \frac{f_y}{f_m'} \right)} \]

For distributed reinforcement, \( \rho \) is the total area of tension reinforcement divided by \( bd \).
Design: Distributed Reinforcement

Calculate

\[ k = \frac{M + P \frac{d_v}{6}}{\frac{1}{3} d_v F_b t_{sp} - P \frac{d_v}{3}} \]

Is \( k \geq k_{bat} \)?

\[ A_{s,reqd} = \frac{\frac{1}{2} k d_v F_b t_{sp} - P}{\frac{1}{2} \frac{(1 - k)^2}{k} d_v n F_b} \]

\[ A_s^* = \text{distributed reinforcement (in.}^2\text{/ft)} \]

Determine \( kd \) from the quadratic equation

\[
\frac{1}{3} d_v^2 F_s t_{sp} n + P \frac{d_v}{3} k^2 + \left[ M - P \frac{d_v}{6} \right] k
- \left[ M + P \frac{d_v}{6} \right] = 0
\]

Solve for \( A_{s,reqd}^* \)

\[
A_{s,reqd}^* = \frac{\frac{1}{2} k d_v t_{sp} F_s \left( \frac{k}{1 - k} \right) \frac{1}{n} - P}{\frac{1}{2} (1 - k) F_s d_v}
\]

Tension controls

Example: Shear Wall

Given: 10 ft high x 16 ft long 8 in. CMU shear wall; Grade 60 steel, Type S mortar; \( f'_m = 2000 \) psi; superimposed dead load of 1 kip/ft. In-plane seismic load of 50 kips. \( S_{DS} = 0.5^\circ \) (just less than 0.5)

Required: Design the shear wall; ordinary reinforced shear wall

Solution: Check using 0.6D+0.7E load combination.

- \( M = 0.7 (50k)(10ft) = 350k \cdot ft = 4200k \cdot in. \)
- Axial load, \( P \)
  - Need to know weight of wall to determine \( P \).
  - Need to know reinforcement spacing to determine wall weight
  - Estimate wall weight as 45 psf
    - Wall weight: 45psf(10ft)(16ft) = 7.2k
  - \( D = 1 \) k/ft (16ft) + 7.2k = 23.2k
  - \( P = (0.6 - 0.7(0.2)S_{DS})D = 0.53D = 0.53(23.2k) = 12.3k \)
**Example: Shear Wall**

Calculate \( k \); for preliminary design purposes use full thickness of wall

\[
k = \frac{M + P \frac{d_v}{6}}{\frac{1}{3} d_v^2 F_b t_{sp} - P \frac{d_v}{3}} = \frac{4200 \text{k in.} + 12.3 \text{k} \frac{192 \text{in.}}{6}}{\frac{1}{3} (192 \text{in.})^2 (0.90 \text{ksi}) (7.625 \text{in.}) - 12.3 \text{k} \frac{192 \text{in.}}{3}} = 0.0550
\]

Since \( k < k_{bal} \) tension controls. Solve quadratic equation.

\[
\left[\frac{1}{3} d_v^2 F_s \frac{t_{sp}}{n} + P \frac{d_v}{3}\right] k^2 + \left[M - P \frac{d_v}{6}\right] k - \left[M + P \frac{d_v}{6}\right] = 0
\]

\[
\left[\frac{1}{3} (192 \text{in.})^2 (32 \text{ksi}) \frac{7.625 \text{in.}}{16.11} + 12.3 \text{k} \frac{192 \text{in.}}{3}\right] k^2 + \left[4200 \text{k in.} - 12.3 \text{k} \frac{192 \text{in.}}{6}\right] k - \left[4200 \text{k in.} + 12.3 \text{k} \frac{192 \text{in.}}{6}\right] = 0
\]

\( k = 0.147 \)

**Example: Shear Wall**

Calculate required area of reinforcement

\[
A_{s, reqd} = \frac{\frac{1}{2} k d_v t_{sp} F_s \left(\frac{k}{1 - k}\right) \frac{1}{n} - P}{\frac{1}{2} (1 - k) F_s d_v}
\]

\[
= \frac{\frac{1}{2} (0.147)(192 \text{in.})(7.625 \text{in.})(32 \text{ksi}) \left(\frac{0.147}{1 - 0.147}\right) \frac{1}{16.11} - 12.3 \text{k}}{\frac{1}{2} (1 - 0.147)(32 \text{ksi})(192 \text{in.})}
\]

\[
= 0.00934 \text{ in.}^2 / \text{in.} = 0.112 \text{ in.}^2 / \text{ft}
\]

Try #5 @ 32 in. (0.120 in.²/ft)

Due to module; use 40 in. (0.093 in.²/ft) for interior bars

**Strength Design:** #4 @ 48 in., 54% of area of steel
Example: Shear Wall

Allowable Values

- Applied Loads
- Balanced Point

Stressed to 88% of allowable

Example: ASD vs. Strength Design

- 0.7φSD: #4 @ 48 in.
- ASD: #5 @ 32,40 in.
Example: Shear Wall

Net area, \( A_{nv} \)

\[ A_{nv} = 2(1.25\text{in.})(192\text{in.}) + 6(8\text{in.})(7.625\text{in.} - 2.5\text{in.}) = 726\text{in.}^2 \]

Shear Stress:

\[ f_v = \frac{V}{A_{nv}} = \frac{0.7(50\text{k})}{726\text{in.}^2} = 48.2\text{psi} \]

Shear Span:

\[ \frac{M}{V_{dv}} = \frac{Vh}{V_{dv}} = \frac{h}{d} = \frac{120\text{in.}}{192\text{in.}} = 0.625 \]

Max Shear:

\[ F_{v,max} = \frac{2}{3} \left( 5 - 2 \frac{M}{V_{dv}} \right) \sqrt{f_m'} y_g \]

\[ = \frac{2}{3} (5 - 2(0.625)) \sqrt{2000\text{psi}} \times 0.75 \approx 83.8\text{psi} \]

Masonry Shear:

\[ F_v = (F_{vm}) y_g = \frac{1}{2} \left[ \left( 4 - 1.75 \frac{M}{V_{dv}} \right) \sqrt{f_m'} + 0.25 \frac{P}{A_n} \right] y_g \]

\[ = \frac{1}{2} \left[ \left( 4 - 1.75(0.625) \right) \sqrt{2000\text{psi}} + 0.25 \frac{12300\text{lb}}{726\text{in.}^2} \right] 0.75 \]

\[ = 51.9\text{psi} \]

Example: Special Reinforced Shear Wall

Given: 10 ft high x 16 ft long 8 in. CMU shear wall; Grade 60 steel, Type S mortar; \( f_m' = 2000 \text{psi} \); superimposed dead load of 1 kip/ft. In-plane seismic load of 50 kips. \( S_{DS} = 0.5 \)

Required: Design the shear wall; special reinforced shear wall

Solution:

• Flexural reinforcement remains the same (although ASCE 7 allows a load factor of 0.9 for ASD and special shear walls)

• Design for 1.5V, or 1.5(0.7)(50 kips) = 52.5 kips (Section 7.3.2.6.1.2)

• \( f_v = 52.5\text{k}/726\text{in.}^2 = 72.3\text{psi} \)

• Maximum \( F_v = 83.8\text{psi} \) OK
Example: Special Reinforced Shear Wall

Masonry Shear:
\[
F_{vm} = \frac{1}{4} \left[ (4 - 1.75 \left( \frac{M}{Vd_v} \right)) \sqrt{f_m'} \right] + 0.25 \frac{P}{A_n}
\]

\[
= \frac{1}{4} \left[ (4 - 1.75(0.625)) \sqrt{2000 \text{psi}} \right] + 0.25 \frac{12300 \text{lb}}{726 \text{in.}^2} = 36.7 \text{psi}
\]

Required steel stress
\[
F_{vs,reqd} = \frac{f_v}{\gamma_g} - F_{vm} = \frac{72.3 \text{psi}}{0.75} - 36.7 \text{psi} = 59.7 \text{psi}
\]

Use #5 bars in bond beams.
Determine spacing.
\[
F_s = 0.5 \left( \frac{A_v F_s d_v}{A_{nv} s} \right) \Rightarrow s = \frac{0.5 A_v F_s d_v}{F_{vs,reqd} A_{nv}}
\]

\[
s = \frac{0.5(0.31 \text{in.}^2)(32000 \text{psi})(192 \text{in.})}{(59.7 \text{psi})(726 \text{in.}^2)} = 21.9 \text{in.}
\]

Use #5 @ 16 in.

Due to closely spaced bond beams, **fully grout wall**.

Shear Area: \( A_{nv} = 7.625 \text{in.} (192 \text{in.}) = 1464 \text{in.}^2 \)

Shear Stress: \( f_v = \frac{V}{A_{nv}} = \frac{52.5 \text{k}}{1464 \text{in.}^2} = 35.9 \text{psi} \)

Wall weight: \( 81 \text{psf}(10 \text{ft})(16 \text{ft}) = 13.0 \text{k} \)

Dead load: \( D = 1 \text{ k/ft} (16 \text{ ft}) + 13.0 \text{k} = 29.0 \text{k} \)

Axial load: \( P = 0.53 D = 0.53 (29.0 \text{k}) = 15.3 \text{k} \)

Masonry Shear:
\[
F_{vm} = \frac{1}{4} \left[ (4 - 1.75 \left( \frac{M}{Vd_v} \right)) \sqrt{f_m'} \right] + 0.25 \frac{P}{A_n}
\]

\[
= \frac{1}{4} \left[ (4 - 1.75(0.625)) \sqrt{2000 \text{psi}} \right] + 0.25 \frac{15300 \text{lb}}{1464 \text{in.}^2} = 35.1 \text{psi}
\]
Example: Special Reinforced Shear Wall

Required steel stress

\[ F_{vs,reqd} = \frac{f_v}{\gamma_g} - F_{vm} = \frac{35.9}{1.0} - 35.1 = 0.8 \text{psi} \]

Use \#4 bars in bond beams. Determine spacing.

\[ F_{vs} = 0.5 \left( \frac{A_v F_se_v}{A_{nv}s} \right) \Rightarrow s = \frac{0.5 A_v F_se_v}{F_{vs,reqd} A_{nv}} \]

\[ s = \frac{0.5(0.20 \text{ in.}^2)(32000 \text{ psi})(192 \text{ in.})}{(0.8 \text{ psi})(1464 \text{ in.}^2)} = 524 \text{ in.} \]

Spacing determined by prescriptive requirements

Maximum Spacing Requirements (7.3.2.6)

- minimum\{ one-third length, one-third height, 48 in. \}

\[ s_{max} = \min \left\{ \frac{192 \text{ in.}}{3}, \frac{120 \text{ in.}}{3}, 48 \text{ in.} \right\} = \min \{64 \text{ in.}, 40 \text{ in.}, 48 \text{ in.} \} = 40 \text{ in.} \]

Example: Special Reinforced Shear Wall

Prescriptive Reinforcement Requirements (7.3.2.6)

- \( \rho \geq 0.0007 \) in each direction
- \( \rho_v + \rho_h \geq 0.002 \)

Vertical: \( \rho_v = \frac{6(0.31 \text{ in.}^2)}{1464 \text{ in.}^2} = 0.00127 \quad \text{OK} \)

Horizontal: \( \rho_{h,reqd} = \max \{0.002 - 0.00127, 0.0007\} = 0.00073 \)

Determine bar size for 40 in. spacing

\[ A_{s,reqd} = \rho_h t_{sp} s = 0.00073(7.625 \text{ in.})(40 \text{ in.}) = 0.22 \text{ in.}^2 \]

Use \#5 @ 40 in.

An alternate is \#4 @ 32 in. (\( \rho_h = 0.00082 \))
Example: Special Reinforced Shear Wall

**Section 8.3.4.4 Maximum Reinforcement**

Requirements only apply to special reinforced shear walls.

No need to check maximum reinforcement since only need to check if:

- \( M/(Vd_v) \geq 1 \) and \( M/(Vd_v) = 0.625 \)
- \( P > 0.05f'_m A_n \)
  - \( 0.05(2000\text{psi})(1464\text{in}^2) = 146 \text{ kips} \)
  - Assume a live load of 1 k/ft
- \( P = (1.0 + 0.7(0.2)S_{DS}) + L = (1.0 + 0.7(0.2)0.5)29k + 16k = 47.0k \)

**OK**

Example: Special Reinforced Shear Wall

If we needed to check maximum reinforcing, do as follows.

\[
\rho_{max} = \frac{nf'_m}{2f_y(n + \frac{f_y}{f'_m})} = \frac{16.1(2\text{ksi})}{2(60\text{ksi})\left(16.1 + \frac{60\text{ksi}}{2\text{ksi}}\right)} = 0.00582
\]

For distributed reinforcement, the reinforcement ratio is obtained as the total area of tension reinforcement divided by \( bd \). For Assume 5 out of 6 bars in tension.

\[
\rho = \frac{A_s}{bd} = \frac{5(0.31\text{in.}^2)}{7.625\text{in.}(188\text{in.})} = 0.00108 \quad \text{OK}
\]