5.1.2 Effective compressive width per bar

5.2 Beams
   5.2.1 General beam design
   5.2.2 Deep beams

9.1.4 Strength reduction factors
9.1.9 Material properties
9.3 Reinforced masonry
   9.3.2 Design assumptions
      9.3.3.2 Maximum area of flexural tensile reinforcement
   9.3.4 Nominal strength
      9.3.4.1.1 Nominal axial and flexural strength
      9.3.4.1.2 Nominal shear strength

9.3.4.2 Beams
   9.3.4.2.1 axial compressive load \( \leq 0.05A_n f_m' \)
   9.3.4.2.2 Longitudinal reinforcement
   9.3.4.2.3 Transverse reinforcement
   9.3.4.2.4 Construction

Design Assumptions

1. Member is straight prismatic section (not in code, but an assumption for our analysis)
2. Plane sections remain plane
3. All masonry in tension is neglected
4. Perfect bond between steel and grout
5. Maximum useable compression strain of masonry
   A. clay masonry: \( \varepsilon_{mu} = 0.0035 \)
   B. concrete masonry: \( \varepsilon_{mu} = 0.0025 \)
6. Equivalent rectangular stress block
   A. Masonry stress = \( 0.8f_m' \)
   B. Masonry stress acts over \( a = 0.8c \)
Beam Design Criteria

5.2.1.3: Length of bearing of beams shall be a minimum of 4 in.; typically assumed to be 8 in.

5.2.1.1.1: Span length of members not built integrally with supports shall be taken as the clear span plus depth of member, but need not exceed distance between center of supports.

5.2.1.2: Compression face of beams shall be laterally supported at a maximum spacing of
   - 32 multiplied by the beam thickness
   - $120b^2/d$

9.3.4.2.4: Beams shall be fully grouted.

9.3.4.2.2.1 The variation in longitudinal reinforcing bars in a beam shall not be greater than one bar size. Not more than two bar sizes shall be used in a beam.

Lintels - Loading

Floor Loading (neglect due to arching)

Floor Loading (include full floor loading)

4 in. (typ)

Opening

Span

45 deg
Lintels - Arching

(State College, PA)

Beam Depth
Beam Construction

Bars placed in bottom U-shaped unit, or knockout bond beam unit.

http://wirebond.com/?page_id=7214

Support on chairs

Wire to web

http://blueinxco.com/Portals/0/Concrete%20Reinforcement%20Products.pdf

Reinforced Masonry - Flexural Members 10

Flexural Members - Strength Design

\[ \varepsilon_{mu} \]

\[ \varepsilon_s \]

\[ \sigma \]

\[ C = \]

\[ T = \]

\[ M_n = \]

\[ \phi = \]

Reinforced Masonry - Flexural Members 11
Reinforced Masonry - Flexural Members

Beams – Behavior

- Masonry Cracks
- Steel Yields
- Masonry Crushes
- Ductile
- Brittle

Minimum reinforcement: (9.3.4.2.2.2, 9.3.4.2.2.3)
- \( M_n \geq 1.3 \times \text{cracking strength} \)
  - Modulus of rupture: Table 9.1.9.2
  - \( M_{cr} = f_r \frac{bh^2}{6} \)
  - or \( A_s \geq \frac{4}{3} A_{s,reqd} \)

Example: 8 inch lintel
- \( h = 7.625 \text{ in.} \)
- \( d = 4 \text{ in.} \)
- \( f_r^\prime = 2000 \text{ psi} \)
- Grade 60 steel
- Type S PCL mortar
- \( f_r = 267 \text{ psi} \)

\[
\begin{array}{|c|c|c|c|}
\hline
& \#3 & \#4 & \#5 \\
\hline
M_{cr} \text{ (k-ft)} & 1.64 & 1.64 & 1.64 \\
1.3M_{cr} \text{ (k-ft)} & 2.14 & 2.14 & 2.14 \\
M_n \text{ (k-ft)} & 2.05 & 3.51 & 5.02 \\
p & 0.00361 & 0.00656 & 0.01016 \\
\rho_{\text{max}} & 0.00952 & 0.00952 & 0.00952 \\
\hline
\end{array}
\]

Minimum reinforcement satisfied if
\[
A_{s,reqd} \leq \frac{3}{4} (0.11) = 0.083 \text{in.}^2
\]
### Modulus of Rupture, psi  Table 9.1.9.2

<table>
<thead>
<tr>
<th>Masonry Type</th>
<th>Mortar Type</th>
<th>Portland cement/lime or mortar cement</th>
<th>Masonry Cement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M or S</td>
<td>N</td>
<td>M or S</td>
</tr>
<tr>
<td>Normal to Bed Joints</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid Units</td>
<td>133</td>
<td>100</td>
<td>80</td>
</tr>
<tr>
<td>Hollow Units*</td>
<td>84</td>
<td>64</td>
<td>51</td>
</tr>
<tr>
<td>Ungrouted</td>
<td>163</td>
<td>158</td>
<td>153</td>
</tr>
<tr>
<td>Fully Grouted</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parallel to bed joints in running bond</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid Units</td>
<td>267</td>
<td>200</td>
<td>160</td>
</tr>
<tr>
<td>Hollow Units</td>
<td>167</td>
<td>127</td>
<td>100</td>
</tr>
<tr>
<td>Ungrouted and partially grouted</td>
<td>267</td>
<td>200</td>
<td>160</td>
</tr>
<tr>
<td>Fully grouted</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parallel to bed joints not laid in running bond</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continuous grout section parallel to bed joints</td>
<td>335</td>
<td>335</td>
<td>335</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

* Use linear interpolation for partially grouted masonry.

### Beams –Maximum Reinforcement

**Maximum reinforcement:** (9.3.3.2)
Reinforcement shall exceed area required to maintain equilibrium under a strain gradient of $\varepsilon_{mu}$ in masonry and $\alpha \varepsilon_y = \_\_\_\_\_ \varepsilon_y$ in reinforcement.

$$\rho_{max} = \frac{0.8 (0.8) f'_m}{f_y} \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right)$$

<table>
<thead>
<tr>
<th>Steel Ratio</th>
<th>Grade 60 steel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Clay</td>
</tr>
<tr>
<td>$\rho_{max}$</td>
<td>0.00565$f'_m$</td>
</tr>
<tr>
<td></td>
<td>0.843$\rho_{max}$</td>
</tr>
<tr>
<td>$f_y = 60$ ksi; $f'_m = 2.00$ ksi</td>
<td>0.01131</td>
</tr>
</tbody>
</table>

$f'_m$ in ksi
Beams – Maximum Reinforcement

**Strain**

\[ \varepsilon_{mu} \]

\[ \alpha \varepsilon_y \]

\[ d \]

\[ \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \]

\[ T = C \]

\[ A_s f_y = 0.8(0.8)f_m' b \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) d \]

\[ \frac{A_s}{bd} = \frac{0.8(0.8)f_m'}{f_y} \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \]

Maximum reinforcement: (9.3.3.2): \( \alpha \varepsilon_y = 1.5 \varepsilon_y \)

Beams – Strength Design Procedure

1. Determine material properties \((f_y, f_m')\)
2. Choose beam dimensions
   - A. Thickness: 8 in., 12 in.
   - B. Depth: if possible, choose so no shear reinforcement is required
3. Determine \(a\), depth of compressive stress block

\[ a = d - \sqrt{d^2 - \frac{2M_n}{0.8f_m'b}} = d - \sqrt{d^2 - \frac{2M_u}{0.8f_m'b}} \]

4. Solve for \(A_{s, reqd}\)

\[ A_{s, reqd} = \frac{0.8f_m'ba}{f_y} \]
Example – Masonry Beam

Given: $M = 150$ k-in. dead load; $M = 150$ k-in. live load; Grade 60 steel, $f_{m} = 2000$ psi; 8 in. CMU; depth of section limited to three courses; Type S masonry cement mortar

Required: Design section

Solution: For three units, $d = 2(8 \text{ in.}) + 4 \text{ in.} = 20 \text{ in.}$

Use \[ A_s = \quad \]
Partially Grouted Walls

*Image*

\[ b = \text{effective compressive width per bar} = \min\{s, 6t, 72 \text{ in.}\} \ (5.1.2) \]
\[ t = \text{nominal thickness} \]

A. Neutral axis in flange:
   a. Almost always the case
   b. Design for solid section

B. Neutral axis in web
   a. Design as a T-beam section

C. Often design based on a 1 ft width

Minimum reinforcement: No requirements

Maximum reinforcement: Same requirement as beams

Partially Grouted Walls: Design Aid

<table>
<thead>
<tr>
<th>Spacing (inches)</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.16</td>
<td>0.30</td>
<td>0.46</td>
<td>0.66</td>
</tr>
<tr>
<td>16</td>
<td>0.082</td>
<td>0.15</td>
<td>0.23</td>
<td>0.33</td>
</tr>
<tr>
<td>24</td>
<td>0.055</td>
<td>0.10</td>
<td>0.16</td>
<td>0.22</td>
</tr>
<tr>
<td>32</td>
<td>0.041</td>
<td>0.075</td>
<td>0.12</td>
<td>0.16</td>
</tr>
<tr>
<td>40</td>
<td>0.033</td>
<td>0.060</td>
<td>0.093</td>
<td>0.13</td>
</tr>
<tr>
<td>48</td>
<td>0.028</td>
<td>0.050</td>
<td>0.078</td>
<td>0.11</td>
</tr>
<tr>
<td>56</td>
<td>0.024</td>
<td>0.043</td>
<td>0.066</td>
<td>0.094</td>
</tr>
<tr>
<td>64</td>
<td>0.021</td>
<td>0.038</td>
<td>0.058</td>
<td>0.082</td>
</tr>
<tr>
<td>72</td>
<td>0.018</td>
<td>0.033</td>
<td>0.052</td>
<td>0.073</td>
</tr>
<tr>
<td>80</td>
<td>0.016</td>
<td>0.030</td>
<td>0.046</td>
<td>0.066</td>
</tr>
<tr>
<td>88</td>
<td>0.015</td>
<td>0.027</td>
<td>0.042</td>
<td>0.060</td>
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<tr>
<td>96</td>
<td>0.014</td>
<td>0.025</td>
<td>0.039</td>
<td>0.055</td>
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<tr>
<td>104</td>
<td>0.013</td>
<td>0.023</td>
<td>0.036</td>
<td>0.051</td>
</tr>
<tr>
<td>112</td>
<td>0.012</td>
<td>0.021</td>
<td>0.033</td>
<td>0.047</td>
</tr>
<tr>
<td>120</td>
<td>0.011</td>
<td>0.020</td>
<td>0.031</td>
<td>0.044</td>
</tr>
</tbody>
</table>
Example - Partially Grouted Walls

Given: 8 in. CMU wall; 12 ft high; Grade 60 steel, $f'_{m} = 2000$ psi; Wind load of $w_{u} = 30$ psf

Required: Reinforcing (place in center of wall)

Solution:

Determine $M_{u}$

\[
M_{u} = \frac{w_{u}h^{2}}{8} = \frac{(30 \text{ lb/ft}^{2})(12 \text{ in.})(12 \text{ ft})^{2}}{8} = 6480 \text{ lb} \cdot \text{in.} = 540 \text{ lb} \cdot \text{ft}
\]

Use # __ @ __ inches ($A_{s}$=____ in.$^{2}$/ft)

Reinforced Masonry - Flexural Members

Example - Partially Grouted Walls

Problem: 72 inch is greater than effective compression width of $6t = 48$ in.

Solution: Use reduced value of $b$: $b = \frac{48 \text{ in.} \cdot 12 \text{ in.}}{72 \text{ in.}} = 8 \text{ in./ft}$

\[
a = d - \sqrt{d^{2} - \frac{2M_{u}}{0.8f'_{m}b}}
\]

Determine $a$

(Assume solid section)

\[
a = 3.81 \text{ in.} - \sqrt{(3.81 \text{ in.})^{2} - \frac{2 \left( \frac{0.540 \text{ kft}}{0.9} \right)(12 \text{ in.})}{0.8 (2.0 \text{ ksi})(8 \text{ in.})}} = 0.1505 \text{ in.}
\]

Determine $A_{s,reqd}$

\[
A_{s,reqd} = \frac{0.8f'_{m}ba}{f_{y}} = \frac{0.8 (2.0 \text{ ksi}) \cdot (8 \text{ in.}) (0.1505 \text{ in.})}{60 \text{ ksi}} = 0.0321 \text{ in.}^{2}/\text{ft}
\]

0.7% increase in required area of steel
- Do not worry about it
- If worried, use $b = 6$ in./ft in design, which is close enough
Example - Partially Grouted Walls

Horizontal spanning masonry between bars:

- Some treat as unreinforced masonry, although debate as to whether you can mix unreinforced and reinforced masonry.
- There is arching occurring, so not truly a simply supported flexural member between vertical bars.
- Can use joint reinforcement

<table>
<thead>
<tr>
<th>Wire Size</th>
<th>$d_b$ (in.)</th>
<th>$A_s$ (in.$^2$)</th>
<th>Spacing (in.)</th>
<th>$\phi M_u$ (kip-ft/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1.7 (9 gage)</td>
<td>0.148</td>
<td>0.017</td>
<td>8</td>
<td>0.921</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>16</td>
<td>0.462</td>
</tr>
<tr>
<td>W2.8 (3/16)</td>
<td>0.187</td>
<td>0.028</td>
<td>8</td>
<td>1.506</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>16</td>
<td>0.841</td>
</tr>
</tbody>
</table>

$\phi = 70$ ksi; $f_m' = 2000$ psi; $d = t_{sp} - 5/8 - d_b/2$

Example: 30 psf; 72 in. spacing; $M_u = 0.135$ k-ft/ft

Partially Grouted Walls - Tolerances

Placement tolerances: (3.4.B.11)

- $d \leq 8$ in. ± 1/2 in.
- 8 in. $< d \leq 24$ in. ± 1 in.
- $d > 24$ in. ± 1 1/4 in.

Along wall: ± 2 in.

8 in. CMU; $f_m' = 2000$ psi; Grade 60
Out-of-Plane: Maximum Reinforcement

8 in. CMU wall, fully grouted, bars in the center, Grade 60 steel, $f_m' = 2000$ psi.
Following table lists the maximum reinforcement for various axial loads.

<table>
<thead>
<tr>
<th>$P / (bd f_m')$</th>
<th>$A_s$ (in.$^2$ per ft)</th>
<th>Spacing (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>#4</td>
</tr>
<tr>
<td>0</td>
<td>0.436</td>
<td>8 (5.5)</td>
</tr>
<tr>
<td>0.05</td>
<td>0.359</td>
<td>8 (6.7)</td>
</tr>
<tr>
<td>0.10</td>
<td>0.283</td>
<td>16 (8.5)</td>
</tr>
<tr>
<td>0.15</td>
<td>0.207</td>
<td>16 (11.6)</td>
</tr>
<tr>
<td>0.20</td>
<td>0.130</td>
<td>24 (18.4)</td>
</tr>
<tr>
<td>0.25</td>
<td>0.054</td>
<td>48 (44.3)</td>
</tr>
</tbody>
</table>

Requirements can be difficult to meet for heavily reinforced shear walls and at jambs.

Shear - Strength Design (9.3.4.1.2)

$$V_n = (V_{nm} + V_{ns}) \gamma_g$$

$$V_{nm} = \left[ 4 - 1.75 \left( \frac{M_u}{V Ud_v} \right) \right] A_{nv} \sqrt{f_m'}$$

$$V_{ns} = 0.5 \left( \frac{A_d}{S} \right) f_y d_v$$

- $\gamma_g$ = partially grouted shear wall factor ($\gamma_g = 1.0$ for beams)
- $d_v$ = actual depth of masonry
- $A_{nv}$ = net shear area = $bd_v$
  - Many designers use $d$ instead of $d_v$ for beams
- $M_u / (V Ud_v)$ need not be taken > 1.0
- $\phi = 0.8$
- Maximum $V_n$
  - $V_n \leq 6A_{nv} \sqrt{f_m'}$ where $\frac{M_u}{V Ud_v} \leq 0.25$
  - $V_n \leq 4A_{nv} \sqrt{f_m'}$ where $\frac{M_u}{V Ud_v} \geq 0.25$
  - Linearly interpolate between 0.25 and 1.0

Conservative approximation

$$M_u / (V Ud_v) = 1.0$$

$$V_{nm} = 2.25A_{nv} \sqrt{f_m'}$$
Shear – Transverse Reinforcement (9.3.4.2.3)

Detailing of shear reinforcement for beams:

a) Single bar with 180-degree hook at each end
b) Hook shear reinforcement around flexural reinforcement
c) Minimum area of shear reinforcement is 0.0007bd

d) First bar within \( \frac{d_v}{4} \)
e) Maximum spacing is \( \frac{d_v}{2} \) or 48 in.

c) Reasonable interpretation is 0.0007bd over \( d_v \).
   This becomes \( \frac{A_v}{s} \geq 0.0007b \)
   ACI 318: \( \frac{A_v}{s} \geq 50 \frac{b}{f_y} = 0.00083b \) for Grade 60 steel

Sections within \( d/2 \) from face of support can be designed for shear at \( d/2 \)
(8.3.5.4): (moved to Chapter 5 in 2022 TMS 402)

A. Noncantilever beam
B. Reaction introduces compression into end region of member
C. No concentrated load between \( \frac{d}{2} \) and face of support

Deflections

- Deflection of beam or lintels supporting unreinforced masonry is limited to \( \frac{L}{600} \) (5.2.1.4.1)
- Deflections of approximately \( L/300 \) needed to be visible.
- Deflections do not need to be checked when \( L \leq 8d \) (5.2.1.4.3).
- End restraint of masonry beams reduces deflections from 20 to 45% of those of simply supported beams.

\[
I_{ef} = I_n \left( \frac{M_{cr}}{M_a} \right) + I_{cr} \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] \quad \text{(Equation 5-1)}
\]

\[
I_{cr} = \frac{bk^3d^3}{3} + nA_s(d - kd)^2 \quad \text{Singly reinforced beam}
\]

\[
I_{cr} = \frac{bk^3d^3}{3} + nA_s(d - kd)^2 + (n - 1)A'_s(kd - d)^2 \quad \text{Doubly reinforced beam}
\]
Flexural Members – Determination of $k$

$$\rho = \frac{A_s}{bd} \quad n = \frac{E_s}{E_m} \quad k = \sqrt{(np)^2 + 2np - np}$$

Doubly reinforced beam

$$k = \sqrt{(n\rho)^2 (1 + r)^2 + 2np \left( \frac{1 + rd'}{d} \right) - n\rho (1 + r)} \quad r = \frac{(n - 1)A_s'}{nA_s}$$

Example: Beam

Given: 10 ft. opening; dead load of 1.5 kip/ft; live load of 1.5 kip/ft; 24 in. high; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; $f'_m = 2000$ psi

Required: Design beam

Solution:

Length of bearing: Assume to be 8 in.

Span length: $L = 10 \text{ ft} + 2(4 \text{ in.}) = 10.67 \text{ ft}$

Lateral support of compression face:

- $32b = 32(7.625 \text{ in.}) = 244 \text{ in.} = 20.3 \text{ ft}$
- $120b^2/d = 120(7.625 \text{ in.})^2 / (20 \text{ in.}) = 349 \text{ in.} = 29.1 \text{ ft}$
Example: Beam, Flexure

**Factored Load**

\[ w = 1.2D + 1.6L \]

\[ = 1.2 \left( 1.5 \text{kft} + 0.083 \frac{\text{k}}{\text{ft}^2} (2 \text{ft}) \right) + 1.6 \left( 1.5 \frac{\text{k}}{\text{ft}} \right) = 4.40 \frac{\text{k}}{\text{ft}} \]

**Factored Moment**

\[ M_u = \frac{w_u L^2}{8} = \left( \frac{4.40}{\text{ft}} \right) (10.67\text{ft})^2 = 62.6 \text{k-ft} \]

*Find \( a \)*

Depth of equivalent rectangular stress block

\[ a = d - \sqrt{d^2 - \frac{2M_n}{0.8f_m' b}} \]

\[ = 20\text{in.} - \sqrt{(20\text{in.})^2 - \frac{2 \left( \frac{62.6 \text{k-ft}}{0.9} \right) (12\text{in.})}{0.8(2.0\text{ksi})(7.625\text{in.})}} = 3.78\text{in.} \]

*Find \( A_{s,reqd} \)*

Req'd area of steel

\[ A_{s,reqd} = \frac{0.8f_m' ba}{f_y} = \frac{0.8(2.0\text{ksi})(7.625\text{in.})(3.78\text{in.})}{60\text{ksi}} = 0.77\text{in.}^2 \]

*Use 2 - #6 (\( A_s = 0.88 \text{in}^2 \))*

\[ M_n = 78.5 \text{ k-ft} \]

\[ \phi M_n = 70.6 \text{ k-ft} \]

---

Example: Beam, Flexure

**Alternate Design:**

Creatively place 1-#6 bar in bottom two courses to facilitate construction.

\( d = \) distance to centroid of tension reinforcement; \( d = 18 \text{ in.} \)

*Find \( a \)*

\[ a = d - \sqrt{d^2 - \frac{2M_n}{0.8f_m' b}} \]

\[ = 18\text{in.} - \sqrt{(18\text{in.})^2 - \frac{2 \left( \frac{62.6 \text{k-ft}}{0.9} \right) (12\text{in.})}{0.8(2.0\text{ksi})(7.625\text{in.})}} = 4.32\text{in.} \]

*Find \( A_{s,reqd} \)*

\[ A_{s,reqd} = \frac{0.8f_m' ba}{f_y} = \frac{0.8(2.0\text{ksi})(7.625\text{in.})(4.32\text{in.})}{60\text{ksi}} = 0.878\text{in.}^2 \]

Check that middle layer has yielded

\[ \varepsilon_s = \frac{\varepsilon_{mu}}{c} \left( d_s - c \right) = \frac{0.0025}{5.40\text{in.}} = (16\text{in.} - 5.40\text{in.}) \]

\[ = 0.00491 \geq 0.00207 = \varepsilon_s \]
Example: Beam, Min and Max Reinforcement

**Minimum Reinforcement Check:** \( f_r = 160 \text{ psi} \) (parallel to bed joints in running bond; fully grouted)

Section modulus \[ S_n = \frac{bh^2}{6} \]

Cracking moment \[ M_{cr} = S_n f_r = 732 \text{ in.}^3 \]

Check 1.3\( M_{cr} \)

\[ 1.3M_{cr} = 1.3(9.76 \text{ k} \cdot \text{ft}) = 12.7 \text{ k} \cdot \text{ft} \leq M_n = 78.5 \text{ k} \cdot \text{ft} \]

**Maximum Reinforcement Check:** \( \rho_{max} = 0.00952 \)

\[ \rho = \frac{A_s}{bd} = \frac{0.88 \text{ in.}^2}{(7.625 \text{ in.})(20 \text{ in.})} = 0.00577 \]

Example: Beam, Development

\[ l_{de} = \frac{0.13d_b^2f_yY_g}{K \sqrt{f_m}} \]

\[
\text{Cover} = 1.25 \text{ (face shell thickness)}
+ 0.5 \text{ (coarse grout)}
+ 0.375 \text{ (assumed #3 stirrup)}
= 2.125 \text{ in.}
\]

\[ \gamma = 1.3 \text{ (#6, #7)} \]

\[ K = \min\{\text{masonry cover, clear spacing between adjacent bars, } 9d_b\} \]

Determine \( K \)

\[ K = \min\{2.125, 9(0.75)\} = 2.125 \text{ in.} \]

Determine \( l_{de} \)

\[ l_{de} = \frac{0.13(0.75 \text{ in.})^2(60000 \text{ psi})(1.3)}{2.125 \text{ in.} \sqrt{2000 \text{ psi}}} = 60.0 \text{ in.} = 5.00 \text{ ft} \]
Example: Beam, Shear Diagram

**Shear Diagram**

- Factor of safety for shear: \( \phi V_{nm} = 12.27 \text{kips} \)
- 23.47 kip
- 18.33 kip
- \( d/2 \) from face of support: 2.54 ft
- 1.17 ft

Example: Beam, Shear

Shear at reaction:

\[
V_u = \frac{w_u L}{2} = \left( \frac{1.2 \left( \frac{k}{f_t} + 0.083 \frac{k}{f_t} (2 \text{ft}) + 1.6 \left( \frac{k}{f_t} \right) \right)}{2} \right) (10.67 \text{ft}) = 23.47 \text{k}
\]

- \( d/2 \) from face of support:
  \[
  \left( \frac{20 \text{in.}}{2} + 4 \text{in.} \right) \frac{1 \text{ft}}{12 \text{in.}} = 1.17 \text{ft}
  \]

Design shear force:

\[
V_u = 23.47 \left( \frac{5.33 \text{ft} - 1.17 \text{ft}}{5.33 \text{ft}} \right) = 18.33 \text{k}
\]

Design masonry shear strength:

\[
\phi V_{nm} = \phi 2.25 A_{nv} \sqrt{f_t} = 0.8(2.25)(7.625 \text{in.})(20 \text{in.}) \sqrt{2000 \text{psi}} = 12.27 \text{k}
\]

- Requirement for shear at \( d/2 \) from face of support is in ASD, not SD, but assume it applies

Check max \( V_n \)

\[
\phi (V_n)_{max} = 4 A_{nv} \sqrt{f_t} = 0.8 (4) (152.5 \text{in.}^2) \sqrt{2000 \text{psi}} = 21.82 \text{k}
\]

\[
V_u = 18.33 \text{k} \leq 21.82 \text{k}
\]

OK
Example: Beam, Shear Reinforcement

Req'd $V_{ns}$

$$V_{ns,req} = \frac{V_u - \phi V_{nm}}{\phi} = \frac{18.33 - 12.27}{0.8} = 7.58 \text{kips}$$

Determine $A_v$ for a spacing of 8 in.

$$V_{ns} = 0.5 \left( \frac{A_v}{s} \right) f_y d_v \Rightarrow A_v = \frac{V_{ns} s}{0.5 f_y d_v}$$

$$A_v = \frac{7.58 \text{kips}(8 \text{in.})}{0.5(60 \text{ksi})(20 \text{in.})} = 0.101 \text{in.}^2$$

**Use #3 stirrups**

Determine $d$ so that no shear reinforcement would be required.

$$d = \frac{V_u}{b(\phi 2.25 \sqrt{f_m})} = \frac{18.33k}{7.625 \text{in.}(0.8)(2.25) \sqrt{2000 \text{psi}}} = 29.9 \text{in.}$$

Use 4 courses ($h = 32 \text{in.}$) and inverted bond beam unit to get $d = 30 \text{in.}$

Example: Beam, Shear Details

### 9.3.4.2.3 Transverse reinforcement

When transverse reinforcement is required, the following provisions shall apply:

(a) Transverse reinforcement shall be a single bar with a 180° hook at each end.
(b) Transverse reinforcement shall be hooked around longitudinal reinforcement.
(c) The minimum area of transverse reinforcement shall be $0.0007 bd_v$.
(d) The first transverse bar shall not be located no more than one-fourth of the beam depth, $d_v$, from the end of the beam.
(e) The maximum spacing shall not exceed 1/2 the depth of the beam nor 48 in.

**Intent of provision (c) is area over a length of $d_v$.**

$$\frac{A_v}{s} = \frac{0.11 \text{in.}^2}{8 \text{in.}} = 0.0138 \text{in.} \geq 0.0007 b = 0.007(7.625 \text{in.}) = 0.0053 \text{in.}$$

**Development length**

$$l_{de} = \frac{0.13 d_v^2 f_y Y}{K \sqrt{f_m}} = \frac{0.13(0.375)^2(60000)(1.0)}{\min\{9(0.375), 1.25 + 0.5\} \sqrt{2000}} = 14.0 \text{in.}$$

With hook:

$$l_{de} = 14.0 \text{in.} - 13(0.375 \text{in.}) = 9.1 \text{in.}$$
Example: Beam, Shear Details

Check width:
2(1.25 in.) Face shells
2(0.75 in.) #6 bars
2(0.5 in.) Coarse grout space
2(0.375 in.) #3 stirrups
1.0 in. space between bars
Total 6.75 in. OK

Need stirrups over first
2.54 ft = 31 in.
First stirrup is at 4 in.
Use stirrups over next 32 in.
Use #3 stirrups at 8 in.

Example: Beam, Alternate Shear Details

Check width:
2(1.25 in.) Face shells
2(0.75 in.) #6 bars
2(0.5 in.) Coarse grout space
2(0.375 in.) #3 stirrups
1.0 in. space between bars
Total 6.75 in. OK

Need stirrups over first
2.54 ft = 31 in.
First stirrup is at 4 in.
Use stirrups over next 32 in.
Use #3 stirrups at 8 in.
Example: Beam, Alternate Shear Details

6.1.7.2.2 Pairs of U-stirrups or ties placed to form a closed unit shall be considered properly spliced when length of laps are $1.7l_v$. In grout at least 18 in. deep, such splices with $A_{vf_y}$ not more than 9,000 lb per leg shall be permitted to be considered adequate if legs extend the full available depth of grout.

$$A_{vf_y} = (0.11in.^2)(60000psi) = 6600lb$$

Example: Shear, Sharpen the Pencil

$$V_{nm} = \left[ 4 - 1.75 \left( \frac{M_u}{V_u d_v} \right) \right] A_{nvf_m}$$

Simply supported beam; uniform load

$$V_u = w_u \left[ \frac{L}{2} - x \right]$$

$$M_u = w_u \left[ \frac{L}{2} x - \frac{x^2}{2} \right]$$

$$(\phi V_{ns})_{reqd} = 4.73 \text{ kips at } x = 1.64 \text{ ft}$$

Previous was 7.58 kips; 38% reduction
Example: Beam, Deflections

Check $L/d$ ratio

$L/d = \frac{10.67\text{ft}}{\frac{12\text{in.}}{20\text{in.}}} = 6.4$

Since $L/d < 8$, deflections are OK; check to show process.

ASD load

$w = D + L = \left(1.5 \frac{k}{ft} + 2ft \left(0.083 \frac{k}{ft^2}\right)\right) + 1.5 \frac{k}{ft} = 3.17 \frac{k}{ft}$

Find $M_a$

$M_a = \frac{wl^2}{8} = \frac{3.17 \frac{k}{ft} (10.67\text{ft})^2}{9} = 45.0k \cdot ft$

Modulus ratio

$n = \frac{E_s}{E_m} = \frac{29000\text{ksi}}{900^2\text{ft}^m} = \frac{29000\text{ksi}}{900(2\text{ksi})} = 16.11$

Reinforcement ratio

$\rho = 0.00577 \quad n\rho = 0.09296$

Find $k$

$k = \sqrt{(n\rho)^2 + 2n\rho - n\rho}$

$= \sqrt{(0.0930)^2 + 2(0.0930) - 0.0930} = 0.348$

$kd = 0.348(20\text{in.}) = 6.96\text{in.}$

Example: Beam, Deflections

Net moment of inertia, $I_n$

$I_n = \frac{bh^3}{12} = 7.625\text{in.} \cdot \frac{7.625\text{in.}(24\text{in.})^3}{12} = 8784\text{in.}^4$

Cracked moment of inertia, $I_{cr}$

$I_{cr} = \frac{bk^3d^3}{3} + nA_s(d - kd)^2$

$= \frac{7.625\text{in.}(6.96\text{in.})^3}{3} + 16.11(0.88\text{in.}^2)(20\text{in.} - 6.96\text{in.})^2$

$= 3268\text{in.}^4$

Effective moment of inertia, $I_{eff}$

$I_{eff} = I_n \left(\frac{M_{cr}}{M_a}\right) + I_{cr} \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right]$

$= 8784\text{in.}^4 \left(\frac{9.76k \cdot ft}{45.0k \cdot ft}\right) + 3268\text{in.}^4 \left[1 - \left(\frac{9.76k \cdot ft}{45.0k \cdot ft}\right)^3\right] = 3324\text{in.}^4$

Deflection, $\delta$

$\delta = \frac{5wl^4}{384EI} = \frac{5 \left(3166 \frac{lb}{ft}\right)(10.67\text{ft})^4}{384(1800000\psi l)(3324\text{in.}^4)} = 1728\text{in.}^3 = 0.154\text{in.}$

Allowable $\delta$

$\frac{L}{600} = \frac{10.67\text{ft}}{600} \frac{12\text{in.}}{ft} = 0.213\text{in.}$

OK
Deep Beams, 5.2.2

- Lintels in which there is a large height of masonry above the opening
- Walls not continuously supported (supported on pier foundations)

- Plane sections do not remain plane
- Internal level arm smaller than computed from linear strain distribution

Effective span length, $L_{eff}$, smaller of:
- center-to-center distance between supports
- 1.15 multiplied by the clear span

$$\frac{L_{eff}}{d_v} \leq \begin{cases} 3 & \text{continuous span} \\ 2 & \text{simple span} \end{cases}$$

<table>
<thead>
<tr>
<th>$z$ – internal lever arm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple spans</td>
</tr>
<tr>
<td>Continuous spans</td>
</tr>
<tr>
<td>$1 \leq \frac{L_{eff}}{d_v} &lt; 2$</td>
</tr>
<tr>
<td>$\frac{L_{eff}}{d_v} &lt; 1$</td>
</tr>
</tbody>
</table>

Deep Beams

- Flexural reinforcement
  - distributed flexural reinforcement for half beam depth
  - maximum spacing of one-fifth $d_v$ or 16 in.
  - joint reinforcement can be used as flexural reinforcement
  - horizontal reinforcement anchored to develop yield strength at face of supports

- Shear reinforcement (when required)
  - minimum area of vertical reinforcement is $0.0007bd_v$
  - horizontal shear reinforcement shall have area ≥ half vertical shear reinforcement
  - maximum spacing of shear reinforcement shall be one-fifth $d_v$ or 16 in.

- Total reinforcement: sum of horizontal and vertical reinforcement shall be at least $0.001bd_v$. 

Reinforced Masonry - Flexural Members
Deep Beams – Example 1

Given: 10 ft. opening; 6 ft. deep beam; dead load (including beam) of 3.0 kip/ft; live load of 2.0 kip/ft; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; $f'_m = 2000 \text{ psi}$

Required: Design beam

Solution:

Center-to-center between supports = 10 ft + 2(4 in.) = 10.67 ft
1.15(clear span) = 1.15(10 ft) = 11.5 ft
Effective span length, $l_{eff} = \min(10.67, 11.5) = 10.67$ ft

Factored moment, $M_u$

$$M_u = \frac{1.2(3.0 \frac{kip}{ft}) + 1.6(2.0 \frac{kip}{ft}) (10.67 \text{ ft})^2}{8} = 96.7\text{ k \cdot ft}$$

Span ratio, $l_{eff} / d_v$

$$\frac{l_{eff}}{d_v} = \frac{10.67\text{ ft}}{6\text{ ft}} = 1.78$$

Internal lever arm, $z$

$$z = 0.2(l_{eff} + 2d_v) = 0.2(10.67\text{ ft} + 2(6\text{ ft})) = 4.53\text{ ft}$$

Reinforced Masonry - Flexural Members 52

Deep Beams – Example 1, Flexure

Req'd $A_s$

$$A_{s,reqd} = \frac{M_u/\phi}{zf_v} = \frac{96.7\text{ k \cdot ft} / 0.9}{4.53\text{ ft} / (60\text{ ksi})} = 0.395\text{ in.}^2$$

**Use 2-#4 bars**

Using standard beam theory, $A_{s,reqd} = 0.320\text{ in}^2$ (19% less)

Flexural reinforcement requirements:
- distributed flexural reinforcement for half beam depth
  - need flexural reinforcement over bottom 36 inches.
- maximum spacing of one-fifth $d_v$, or 16 in.
  - $(1/5)(72) = 14.4$ in.
- Use W1.7 (9 gage) joint reinforcement every 8 in. in bottom 5 bed joints (as a practical matter, use in every bed joint in beam)
**Deep Beams – Example 1, Shear**

Factored shear, \( V_u \)

\[
V_u = \frac{1.2\left(3.0\frac{k}{ft}\right) + 1.6\left(2.0\frac{k}{ft}\right)}{2}(10.67\text{ft}) = 36.3\text{kip}
\]

Design shear strength, \( \phi V_{nm} \)

\[
\phi V_{nm} = \phi 2.25A_{nv}\sqrt{f_m^r} = 0.8(2.25)(7.625\text{in.})(68\text{in.})\sqrt{2000\text{psi}} = 41.7\text{k}
\]

No shear reinforcement required

Total reinforcement

\[
0.001bd_v = 0.001(7.625\text{in.})(72\text{in.}) = 0.55\text{ in.}^2
\]

\[
2-\#4 (0.40\text{in.}^2) + 5(2)(0.017\text{in.}^2) = 0.57\text{in.}^2 \quad \text{OK}
\]

Joint reinforcement

**Deep Beams – Example 1, Development**

Clear cover

\[
1.25 \text{ (face shell)} + 0.5 \text{ (coarse grout)} = 1.75 \text{ in.}
\]

Development length, \( l_{de} \)

\[
l_{de} = \frac{0.13d^2_f f_y g}{K\sqrt{f_m^r}} = \frac{0.13(0.5\text{in.})^2(60000\text{psi})(1.0)}{\min\{9(0.5\text{in.}), 1.75\text{in.}\} \sqrt{2000\text{psi}}} = 24.9\text{in.}
\]

Extend bars 24 in. beyond face of support

(close enough; cover of 1.82 in., 1/16 in. more than 1.75 in., results in \( l_{de} = 24 \text{ in.} \))
Deep Beams – Example 2

Given: 10 ft. opening; 6 ft. deep beam; dead load (including beam) of 6.0 kip/ft; live load of 3.0 kip/ft; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; $f'_m = 2000$ psi

Required: Design beam

Solution:

Factored moment, $M_u = \frac{\left[1.2\left(6.0 \frac{k}{ft}\right)+1.6\left(3.0 \frac{k}{ft}\right)\right](10.67 \text{ft})^2}{8} = 170.7k \cdot ft$

Req’d $A_s$ \[ A_s,\text{reqd} = \frac{M_u}{\phi f_y} = \frac{170.7k \cdot ft/0.9}{4.53 \text{ft}(60\text{ksi})} = 0.697 \text{in.}^2 \]

Use 2-#4 bars in each of bottom 2 courses (4 total bars)

Using standard beam theory, $A_s,\text{reqd} = 0.607 \text{in}^2$ (13% less)

Use joint reinforcement in every course for distributed reinforcement

Deep Beams – Example 2, Shear

Factored shear, $V_u = \frac{\left[1.2\left(6.0 \frac{k}{ft}\right)+1.6\left(3.0 \frac{k}{ft}\right)\right](10.67 \text{ft})}{2} = 64.0\text{kip}$

Max shear \[ \phi V_{u,max} = 0.8 \left[4A_{nv}\sqrt{f_m} \right] = 0.8\left[4(518\text{in.}^2)\sqrt{2000\text{psi}} \right] = 74.1\text{kip} \]

Req’d shear strength, $V_{ns}$ \[ \phi V_{ns,reqd} = 64.0k - 41.7k = 22.3k \]

Req’d shear steel, $A_v$ \[ A_v = \frac{V_{ns,reqd}}{0.5f_y d_y} = \frac{(22.3k/0.8)(8\text{in.})}{0.5(60\text{ksi})(72\text{in.})} = 0.103\text{in.}^2 \]

Use 1-#3 at 8 in. vertical shear reinforcement
**Deep Beams – Example 2, Shear Details**

- Minimum area of vertical reinforcement is $0.0007bd_v$
  - $0.0007bd_v = 0.0007(7.625)(72) = 0.384\text{ in.}^2$
  - Total vertical is $0.11\text{ in.}^2(15) = 1.65\text{ in.}^2$

- Horizontal shear reinforcement shall have area ≥ half vertical shear reinforcement
  - Use 1-#4 each course
  - Do not need joint reinforcement

- Maximum spacing of shear reinforcement shall be one-fifth $d_v$ or 16 in.
  - Maximum spacing is 14.4 in.