Combined Flexural and Axial Loads

- Bearing Walls
- Pilasters
- Columns

Combined Flexure and Axial Load

Design Strength \( \geq \) Factored Load

(Interaction Diagram) (Second-order; P-delta)
Key Code Sections

5.3 Columns
5.4 Pilasters
9.3.2 Design assumptions
9.3.4.1 Nominal strength
  9.3.4.1.1 Nominal axial and flexural strength
  Section 4.3.3 Radius of gyration
9.3.5 Wall design for out-of-plane loads
  9.3.5.1 Scope
  9.3.5.2 Nominal axial and flexural strength
  9.3.5.3 Nominal shear strength
  9.3.5.4 P-delta effects
  9.3.5.5 Deflections

Concentric Axial Compression

\[ P_n = 0.80 \left[ 0.80 f_m' (A_n - A_{st}) + f_y A_{st} \right] \left[ 1 - \left( \frac{h}{140r} \right)^2 \right] \text{ for } \frac{h}{r} \leq 99 \]

\[ P_n = 0.80 \left[ 0.80 f_m' (A_n - A_{st}) + f_y A_{st} \right] \left( \frac{70r}{h} \right)^2 \text{ for } \frac{h}{r} > 99 \]

\[ \phi = 0.9 \quad A_{st} = \text{area of laterally tied steel} \]

\[ P_{euler} = \frac{\pi^2 EI}{h^2} = \frac{\pi^2 E A_n r^2}{h^2} = \frac{\pi^2 (900f_m')A_n r^2}{h^2} = A_n f_m' \left( 94.2 \frac{r}{h} \right)^2 \]

- Equation above for CMU; for clay \((E_m = 700 f_m')\), term is \((83.1r/h)^2\)
- Code equation actually derived from unreinforced masonry and a no-tension material, but similar to Euler buckling
**Buckling Curve for $A_{st} = 0$**

\[
P_n = 0.80[0.80 f_m A_n] \left( \frac{795r}{h} \right)^2
\]

\[
P_n = 0.80[0.80 f_m A_n] \left[ 1 - \left( \frac{h}{140r} \right)^2 \right]
\]

**Radius of Gyration**

4.3.3 *Radius of gyration*

Radius of gyration shall be computed using average net cross-sectional area of the member considered.

**Questions:**
- Is this a strict average or weighted average?
- What about different types of units (which changes block area)?
- What is the effect of bond beams?

- NCMA has tabulated values of average radii of gyration based on block area.
- Bennett often uses $r = \sqrt{I_n/A_n}$ in the examples and spreadsheets.
Interaction Diagram

- Assume strain/stress distribution or depth to neutral axis
- Compute forces in masonry and steel
- Sum forces to get axial force
- Sum moment about centerline to get bending moment
- Key points
  - Pure axial load \((M_n = 0)\)
  - Pure bending \((P_n = 0)\)
  - Balanced \((strain \ of \ \varepsilon_{mu} \ and \ \varepsilon_y)\)

Example – 8 in. CMU Bearing Wall

Given: 12 ft high CMU bearing wall, Type S masonry cement mortar; Grade 60 steel in center of wall; #4 @ 48 in.; partial grout; \(f'_{m} = 2000 \text{ psi}\)

Required: Interaction diagram in terms of capacity per foot

**Pure Moment:**

\[
M_n = A_s f_y \left( d - \frac{1}{2} \frac{A_s f_y}{0.8 b f_m'} \right)
\]

Nominal moment, \(M_n\)

Design moment, \(\phi M_n\)

\[
\phi M_n = 0.9 \left( 0.934 \frac{k\cdot ft}{ft} \right) = 0.840 \frac{k\cdot ft}{ft}
\]

Check to make sure stress block is in face shell

\[
a = \frac{A_s f_y}{0.8 b f_m'} = \frac{0.05 \text{in.}^2}{0.8 \left( \frac{12 \text{in.}}{ft} \right) \left( 2.0 \text{ksi} \right)} = 0.156 \text{ in.}
\]
Example – 8 in. CMU Bearing Wall

**Pure Axial:**
NCMA TEK 14-1B  Section Properties of Concrete Masonry Walls
\[ r = 2.66 \text{ in.} \quad A_n = 40.7 \text{in.}^2/\text{ft} \quad I_n = 332.0 \text{ in.}^4/\text{ft} \]

Find \( h/r \)

\[ P_n = 0.80 \left[ 0.80 f_m' (A_n - A_{st}) + f_y A_{st} \right] \left[ 1 - \left( \frac{h}{140r} \right)^2 \right] \]

Nominal Axial Load, \( P_n \)

Design axial load, \( \phi P_n \)

\[ \phi P_n = 0.9 \left( \frac{44.3 \text{ k}}{\text{ft}} \right) = 39.9 \frac{\text{k}}{\text{ft}} \]

Using \( r = \sqrt{\frac{I_n}{A_n}} = \sqrt{\frac{332.0 \text{ in.}^4}{40.7 \text{in.}^2/\text{ft}}} = 2.86 \text{ in.} \) \( h/r = 50.4 \) \( \phi P_n = 40.8 \text{ k/ft} \) 2.2% greater

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**Balanced:**

\[ \varepsilon_{mu} = 3.81 \text{ in.} \quad \varepsilon_y = 2.09 \text{ in.} \]

Strain

Stress

Web length =

\[ c = \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y} d = \frac{0.0025}{0.0025 + 0.000207} = 3.81 \text{ in.} = 2.09 \text{ in.} \]

\[ a = 0.8c = 0.8(2.09 \text{ in.}) = 1.67 \text{ in.} \]
Example – 8 in. CMU Bearing Wall

Balanced:

Compressive force, $C_m$

\[ C_{m,\text{face shell}} = \]
\[ C_{m,\text{web}} = \]

Tension force, $T$

\[ T = f_y A_s = 60 ksi \left( 0.05 \frac{\text{in.}^2}{\text{ft}} \right) = 3.0 \frac{k}{\text{ft}} \]

Design force, $\phi P_n$

\[ \phi P_n = 0.9 (24.0 + 1.3 - 3.0) \frac{k}{\text{ft}} = 20.1 \frac{k}{\text{ft}} \]

Design moment, $\phi M_n$

\[ \phi M_n = \]

\[ \phi P_n = 20.1 \frac{k}{\text{ft}} \]
\[ \phi M_n = 5.97 \frac{k\cdot\text{ft}}{\text{ft}} \]

Example – 8 in. CMU Bearing Wall

Below Balanced:

\[ c = 1.25 \text{ in.} \]
\[ 0.8 f_m' = 1.6 \text{ ksi} \]
\[ \varepsilon_{mu} = 0.0025 \]
\[ 3.81 \text{ in.} \]
\[ 1.25 \text{ in.} \]
\[ 1.0 \text{ in.} \]
\[ \varepsilon_s = \frac{d-c}{c} \varepsilon_{mu} = \frac{3.81 \text{ in.} - 1.25 \text{ in.}}{1.25 \text{ in.}} \times 0.0025 = 0.00512 \]

\[ C_m = [0.8(2.0 ksi)](1.0 \text{ in.}) \left( 12 \frac{\text{in.}}{\text{ft}} \right) = 19.2 \frac{k}{\text{ft}} \]

\[ T = f_y A_s = 60 ksi \left( 0.05 \frac{\text{in.}^2}{\text{ft}} \right) = 3.0 \frac{k}{\text{ft}} \]

\[ \phi P_n = 0.9 (19.2 - 3.0) \frac{k}{\text{ft}} = 14.6 \frac{k}{\text{ft}} \]

\[ \phi M_n = 0.9 \left[ 19.2 \frac{k}{\text{ft}} \left( 3.81 - \frac{1.00}{2} \right) \text{ in.} \right] = 4.77 \frac{k\cdot\text{ft}}{\text{ft}} \]
Example – 8 in. CMU Bearing Wall

Above Balanced:
\[ c = 3.0 \text{ in.} \]
\[ \epsilon_{s} = 0.000677 \]
\[ f_{y} = 0.8 \text{ksi} \]

\[ C_{m,fs} = [0.8(2.0\text{ksi})](1.25\text{in.}) \left( 12 \frac{\text{in.}}{\text{ft}} \right) = 24.0 \frac{\text{k}}{\text{ft}} \]

\[ C_{m,web} = [0.8(2.0\text{ksi})](2.4\text{in.} - 1.25\text{in.}) \left( 2.0 \frac{\text{in.}}{\text{ft}} \right) = 3.68 \frac{\text{k}}{\text{ft}} \]

\[ T = E_{s} \epsilon_{s} A_{s} = 29000\text{ksi}(0.000677) \left( 0.05 \frac{\text{in.}^{2}}{\text{ft}} \right) = 0.98 \frac{\text{k}}{\text{ft}} \]

\[ \phi P_{n} = 0.9(24.0 + 3.68 - 0.98) \frac{\text{k}}{\text{ft}} = 24.0 \frac{\text{k}}{\text{ft}} \]

\[ \phi M_{n} = 0.9 \left[ 24.0 \frac{\text{k}}{\text{ft}} \left( 3.81 - \frac{1.25}{2} \right) \text{in.} + 3.68 \frac{\text{k}}{\text{ft}} \left( 3.81 - 1.5 - \frac{24-1.25}{2} \right) \text{in.} \right] \]

\[ = 6.28 \frac{\text{k} \cdot \text{ft}}{\text{ft}} \]

Combined Flexural and Axial Loads
Example – 8 in. CMU Bearing Wall

Interaction Diagram – Solid vs. Partial Grout
Interaction Diagram Approxiimations

Interaction Diagram – Below Balanced

Tension, $T$  
$T = f_y A_s$

Compression, $C_m$  
$C_m = 0.8 f'_m b a$

Nominal Axial Strength, $P_n$  
$P_n = C_m - T = 0.8 f'_m b a - A_s f_y$

Solve for $a$  
$a = \frac{A_s f_y + P_n}{0.8 f'_m b}$

Nominal Moment Strength, $M_n$  
$M_n = 0.8 f_m b a \left(\frac{t_{sp} - \frac{a}{2}}{2}\right) + A_s f_y \left(\frac{d - \frac{t_{sp}}{2}}{2}\right)$

$= (P_n + A_s f_y) \left(\frac{t_{sp} - \frac{a}{2}}{2}\right) + A_s f_y \left(\frac{d - \frac{t_{sp}}{2}}{2}\right)$

Can solve for $M_n$ if $P_n$ is known
If we could only know one point on the interaction diagram, we would want to know the point corresponding to $\phi P_n = P_u$.

$$a = \frac{A_s f_y + P_u/\phi}{0.8 f_m' b}$$

$$M_n = (P_u/\phi + A_s f_y) \left(\frac{t_{sp} - a}{2}\right) + A_s f_y \left(d - \frac{t_{sp}}{2}\right)$$

These are equations in 9.3.5.2 commentary. They ignore any tension in a possible second layer of steel near the compression face.

For centered bars:

$$M_n = (P_u/\phi + A_s f_y) \left(d - \frac{a}{2}\right)$$

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$t_{sp} - t_{fs} < 0.8c < t_{sp}$

$$C_1 = 0.8f_m' b t_{fs}$$

$$C_2 = 0.8f_m' b_w (t_{sp} - 2t_{fs})$$

$$C_3 = 0.8f_m' b \left(t_{fs} - \left(t_{sp} - a\right)\right)$$

$$P_n = C_1 + C_2 + C_3$$

$$M_n = C_1 x_1 + C_3 x_3$$

$$x_1 = \frac{t_{sp}}{2} - \frac{t_{fs}}{2}$$

$$x_3 = \frac{t_{fs} - a}{2}$$
Interaction Diagram – Partially Grouted Wall

\[ C_1 = 0.8f_m'b_f \]
\[ C_2 = 0.8f_m'b_w(a - t_{fs}) \]
\[ P_n = C_1 + C_2 \]
\[ M_n = C_1x_1 + C_2x_2 \]
\[ x_1 = \frac{t_{sp} - t_{fs}}{2} \]
\[ x_2 = \frac{t_{sp}}{2} - \frac{a}{2} - \frac{t_{fs}}{2} \]
\[ x_T = d - t_{sp} \]

Combined Flexural and Axial Loads
Interaction Diagram – Partially Grouted Wall

\[ 0.8c \leq t_{fs} \]

\[ C_1 = 0.8f'_{mu}ba \]

\[ T = A_s \left[ \min \left\{ f_y, E_s \varepsilon_{mu} \frac{d - c}{c} \right\} \right] \]

\[ P_n = C_1 - T \]

\[ M_n = C_1 x_1 + T x_T \]

\[ x_1 = \frac{t_{sp}}{2} - \frac{a}{2} \]

\[ x_T = d - \frac{t_{sp}}{2} \]

Combined Flexural and Axial Loads

Walls: Maximum Reinforcement

- Strain gradient of \( \varepsilon_{mu} \) and \( \alpha \varepsilon_y \), with \( \alpha = 1.5 \) for OOP loading
- \( P_u \) determined from \( D + 0.75L + 0.525Q_E \)

Fully grouted with equal tension and compression reinforcement

\[
\rho = \frac{A_s}{bd} = \frac{0.64f'_{m} \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - \frac{P_u}{bd}}{f_y - \min \left( \varepsilon_{mu} - \frac{d'}{d} \left( \varepsilon_{mu} + \alpha \varepsilon_y \right), \varepsilon_y \right) E_s}
\]

Fully grouted with concentrated tension reinforcement, or partially grouted with neutral axis in face shell

\[
\rho = \frac{A_s}{bd} = \frac{0.64f'_{m} \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - \frac{P_u}{bd}}{f_y}
\]

Partially grouted walls with concentrated tension reinforcement and neutral axis in web

\[
\rho = \frac{0.64f'_{m} \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \left( \frac{b_w}{b} \right) + 0.8f'_{m} t_{fs} \left( \frac{b - b_w}{bd'} \right) - \frac{P_u}{bd}}{f_y}
\]
### Walls: Maximum Reinforcement

Maximum Axial Load from Load Combination $D + 0.75L + 0.525Q_e$ to Meet Maximum Reinforcement Requirements for 8 in. CMU Wall, Centered Grade 60 Reinforcement, $f'_{m} = 2000$ psi

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>Bar spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8 in.</td>
</tr>
<tr>
<td>No. 4</td>
<td>8.1 k/ft</td>
</tr>
<tr>
<td>No. 5</td>
<td>1.5 k/ft</td>
</tr>
<tr>
<td>No. 6</td>
<td>5.3 k/ft</td>
</tr>
<tr>
<td>No. 7</td>
<td>1.2 k/ft</td>
</tr>
</tbody>
</table>

- For values not listed, a tension force would be required to meet the maximum reinforcement requirements.
- Values listed in **green** are for $f'_{m} = 2250$ psi; require a unit strength of 2600 psi.