TMS 402 Overview

Part 1: General
  Chapter 1: General Requirements
  Chapter 2: Notation and Definitions
  Chapter 3: Quality and Construction

Part 2: Design Requirements
  Chapter 4: General Analysis and Design Considerations
  Chapter 5: Structural Members
  Chapter 6: Reinforcement, Metal Accessories, and Anchor Bolts
  Chapter 7: Seismic Design Requirements

Part 3: Engineered Design Methods
  Chapter 8: Allowable Stress Design of Masonry
  Chapter 9: Strength Design of Masonry
  Chapter 10: Prestressed Masonry
  Chapter 11: Strength Design of Autoclaved Aerated Concrete (AAC) Masonry

Part 4: Prescriptive Design Methods
  Chapter 12: Veneer
  Chapter 13: Glass Unit Masonry
  Chapter 14: Masonry Partition Walls

Appendices:
  Appendix A: Empirical Design of Masonry
  Appendix B: Design of Masonry Infill
  Appendix C: Limit Design Method

TMS 402 Chapter 9 Strength Design of Masonry

9.1 - General
  9.1.1 Scope
  9.1.2 Required strength
  9.1.3 Design strength
  9.1.4 Strength-reduction factors
  9.1.5 Deformation requirements
  9.1.6 Anchor bolts embedded in grout
  9.1.7 Shear strength in multiwythe masonry
  9.1.8 Nominal bearing strength
  9.1.9 Material properties

9.2 - Unreinforced Masonry

9.3 – Reinforced Masonry
  9.3.1 Score
  9.3.2 Design assumptions
  9.3.3 Reinforcement requirements and details
  9.3.4 Design of beams and columns
  9.3.5 Wall design for out-of-plane loads
  9.3.6 Wall design for in-plane loads
Flexural Members - Strength Design

5.1.2 Effective compressive width per bar

5.2 Beams
   5.2.1 General beam design
   5.2.2 Deep beams

9.1.4 Strength reduction factors

9.1.9 Material properties

9.3 Reinforced masonry
   9.3.2 Design assumptions
   9.3.3.2 Maximum area of flexural tensile reinforcement

9.3.4 Nominal strength
   9.3.4.1.1 Nominal axial and flexural strength
   9.3.4.1.2 Nominal shear strength

9.3.4.2 Beams
   9.3.4.2.1 Axial compressive load \( \leq 0.05A_{nf}f'_m \)
   9.3.4.2.2 Longitudinal reinforcement
   9.3.4.2.3 Transverse reinforcement
   9.3.4.2.4 Construction

Design Assumptions

1. Member is straight prismatic section (not in code, but an assumption for our analysis)
2. Plane sections remain plane
3. All masonry in tension is neglected
4. Perfect bond between steel and grout
5. Maximum useable compression strain of masonry
   A. clay masonry: \( \varepsilon_{mu} = 0.0035 \)
   B. concrete masonry: \( \varepsilon_{mu} = 0.0025 \)
6. Equivalent rectangular stress block
   A. Masonry stress = \( 0.8f'_m \)
   B. Masonry stress acts over \( a = 0.8c \)
Flexural Members - Strength Design

\[ M_n = \text{__________} \text{moment strength; calculated strength using nominal values (e.g. } f_y = 60 \text{ ksi)} \]

\[ \phi M_n = \text{__________} \text{moment strength; value used in design} \]

\[ \phi = \text{______} \text{ for flexure (9.1.4.4)} \]

\[ M_u = \text{__________} \text{moment; independent of member except for self-weight} \]

Strength Design (Load and Resistance Factor Design)

\[ \phi M_n \geq M_u \]

Reinforced Masonry - Flexural Members
Beams – Strength Design Procedure

1. Determine material properties \( (f_y, f_m') \)
2. Choose beam dimensions
   A. Thickness: 8 in., 12 in.
   B. Depth: if possible, choose so no shear reinforcement is required
3. Determine \( a \), depth of compressive stress block
   \[
   a = d - \sqrt{d^2 - \frac{2M_n}{0.8f_m' b}} = d - \sqrt{d^2 - \frac{2M_u}{0.8\phi f_m' b}}
   \]
4. Solve for \( A_{s,reqd} \)
   \[
   A_{s,reqd} = \frac{0.8f_m' ba}{f_y}
   \]

Example – Masonry Beam

Given: \( M = 150 \) k-in. dead load; \( M = 150 \) k-in. live load; Grade 60 steel, \( f_m' = 2000 \) psi; 8 in. CMU; depth of section limited to three courses; Type S masonry cement mortar

Required: Design section

Solution: For three units, \( d = 2(8 \text{ in.}) + 4 \text{ in.} = 20 \text{ in.} \)

Use \( A_s = \)
Beams – Behavior

Minimum reinforcement: (9.3.4.2.2.2, 9.3.4.2.2.3)

- \( M_n \geq 1.3M_{cr} \)
  - Modulus of rupture: Table 9.1.9.2
  - \( M_{cr} = f_r \frac{bh^2}{6} \)
  - or \( A_s \geq (4/3)A_{s,reqd} \)

<table>
<thead>
<tr>
<th>Flexural Tension Parallel to Bed Joint</th>
<th>Mortar Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PCL or mortar cement</td>
</tr>
<tr>
<td></td>
<td>M or S</td>
</tr>
<tr>
<td></td>
<td>M or S</td>
</tr>
<tr>
<td>Running bond; hollow units; fully grouted</td>
<td>267</td>
</tr>
<tr>
<td>Not laid in running bond; continuous grout section parallel to bed joints</td>
<td>335</td>
</tr>
</tbody>
</table>

Design Tip: \( M_n \) often not calc \( M_u \geq 1.17M_{cr} \)
Maximum reinforcement: (9.3.3.2)
Reinforcement shall NOT exceed area required to maintain equilibrium under a strain gradient of $\varepsilon_{\text{mu}}$ in masonry and $\alpha \varepsilon_y = \text{_____} \varepsilon_y$ in reinforcement.

$$\rho_{\text{max}} = 0.8 \left(0.8\right) f_m' \left( \frac{\varepsilon_{\text{mu}}}{\varepsilon_{\text{mu}} + \alpha \varepsilon_y} \right)$$

<table>
<thead>
<tr>
<th>Steel Ratio</th>
<th>Grade 60 steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>CMU</td>
</tr>
<tr>
<td>$\rho_{\text{max}}$</td>
<td>0.00565$f_m'$</td>
</tr>
<tr>
<td></td>
<td>0.843$\rho_{\text{bal}}$</td>
</tr>
<tr>
<td>$f_y = 60$ ksi; $f_m' = 2$ ksi</td>
<td>0.00952</td>
</tr>
<tr>
<td>$f_y = 60$ ksi; $f_m' = 3$ ksi</td>
<td>0.01695</td>
</tr>
</tbody>
</table>

$T = C$

$$A_s f_y = 0.8 \left(0.8\right) f_m' b \left( \frac{\varepsilon_{\text{mu}}}{\varepsilon_{\text{mu}} + \alpha \varepsilon_y} \right) d$$

$$A_s \frac{b d}{f_y} = 0.8 \left(0.8\right) f_m' \left( \frac{\varepsilon_{\text{mu}}}{\varepsilon_{\text{mu}} + \alpha \varepsilon_y} \right)$$

Maximum reinforcement: (9.3.3.2): $\alpha \varepsilon_y = 1.5 \varepsilon_y$
Example – Masonry Beam, min & max steel

24 inch high 8 in. CMU beam, $f_m' = 2000$ psi; #6 Grade 60 steel; Type S masonry cement mortar; $M_u = 420k\cdot\text{in.}$

**Check minimum steel**

$$f_r = \text{______ psi} \quad \text{(Type S masonry cement mortar)}$$

Cracking moment: $$M_{cr} = f_r \frac{bh^2}{6} =$$

**Check maximum steel**

$$\rho = \frac{A_s}{bd} = \text{OK}$$

Shear - Strength Design (9.3.4.1.2)

$$V_n = (V_{nm} + V_{ns})\gamma_g$$

$$V_{nm} = \left[ 4 - 1.75 \left( \frac{M_u}{V_u d_v} \right) \right] A_{nv} \sqrt{f_m'}$$

$$V_{ns} = 0.5 \left( \frac{A_v}{s} \right) f_v d_v$$

- $\gamma_g$ = partially grouted shear wall factor ($\gamma_g = 1.0$ for beams)
- $d_v$ = actual depth of masonry
- $A_{nv}$ = net shear area = $bd_v$
  - Many designers use $d$ instead of $d_v$ for beams
- $M_u/(V_u d_v)$ need not be taken $> 1.0$
- $\phi = 0.8$
- **Maximum $V_n$**
  - $V_n \leq 6A_{nv} \sqrt{f_m'}$ where $\frac{M_u}{V_u d_v} \leq 0.25$
  - $V_n \leq 4A_{nv} \sqrt{f_m'}$ where $\frac{M_u}{V_u d_v} \geq 0.25$
  - Linearly interpolate between 0.25 and 1.0

**Conservative approximation**

$$\frac{M_u}{(V_u d_v)} = 1.0$$

$$V_{nm} = 2.25A_{nv} \sqrt{f_m'}$$
### Shear – Transverse Reinforcement (9.3.4.2.3)

Detailing of shear reinforcement for beams:

a) Single bar with 180-degree hook at each end
b) Hook shear reinforcement around flexural reinforcement
c) Minimum area of shear reinforcement is $0.0007bd_v$
d) First bar within $d_v/4$
e) Maximum spacing is $d_v/2$ or 48 in.

c) Reasonable interpretation is $0.0007bd_v$ over $d_v$.
This becomes $\frac{A_v}{s} \geq 0.0007b$
ACI 318: $\frac{A_v}{s} \geq 50\frac{b}{f_y} = 0.00083b$ for Grade 60 steel

Sections within $d/2$ from face of support can be designed for shear at $d/2$
(8.3.5.4): (moved to Chapter 5 in 2022 TMS 402)

A. Noncantilever beam
B. Reaction introduces compression into end region of member
C. No concentrated load between $d/2$ and face of support

### Deflections

- Deflection of beam or lintels supporting unreinforced masonry is limited to $L/600$ (5.2.1.4.1)
- Deflections of approximately $L/300$ needed to be visible.
- Deflections do not need to be checked when $L \leq 8d$ (5.2.1.4.3).
- End restraint of masonry beams reduces deflections from 20 to 45% of those of simply supported beams.

\[
I_{eff} = I_n \left( \frac{M_{cr}}{M_a} \right)^3 + I_{cr} \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right]
\]
(Equation 5-1)

\[
I_{cr} = \frac{bk^3d^3}{3} + nA_s(d - kd)^2
\]  
Singly reinforced beam

\[
I_{cr} = \frac{bk^3d^3}{3} + nA_s(d - kd)^2 + (n - 1)A_s'(kd - d)^2
\]  
Doubly reinforced beam
**Flexural Members – Determination of k**

\[
\rho = \frac{A_s}{bd} \quad n = \frac{E_s}{E_m} \quad k = \sqrt{(nP)^2 + 2nP - nP}
\]

Doubly reinforced beam

\[
k = \sqrt{(nP)^2(1 + r)^2 + 2nP \left(\frac{1 + rd'}{d}\right) - nP(1 + r)}
\]

\[
r = \frac{(n - 1)A_s'}{nA_s}
\]

**Non-Bearing Walls**

Design based on 1 ft strip

[Image of a construction site]

https://vancouverconcrete.net/vancouver-block-walls-installation.html
### Partially Grouted Walls

**Minimum reinforcement:**
No requirements

**Maximum reinforcement:**
Same requirement as beams

A. Neutral axis in flange:
   a. Almost always the case
   b. Design for solid section

B. Neutral axis in web
   a. Design as a T-beam section

C. Often design based on a 1 ft width

\[ b = \text{effective compressive width per bar} = \min\{s, 6t, 72 \text{ in.}\} \]  
\[ t = \text{nominal thickness} \]  

### Partially Grouted Walls: Design Aid

<table>
<thead>
<tr>
<th>Spacing (inches)</th>
<th>Steel Area in.²/ft</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#3</td>
</tr>
<tr>
<td>8</td>
<td>0.16</td>
</tr>
<tr>
<td>16</td>
<td>0.082</td>
</tr>
<tr>
<td>24</td>
<td>0.055</td>
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<tr>
<td>32</td>
<td>0.041</td>
</tr>
<tr>
<td>40</td>
<td>0.033</td>
</tr>
<tr>
<td>48</td>
<td>0.028</td>
</tr>
<tr>
<td>56</td>
<td>0.024</td>
</tr>
<tr>
<td>64</td>
<td>0.021</td>
</tr>
<tr>
<td>72</td>
<td>0.018</td>
</tr>
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<td>80</td>
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<td>88</td>
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<td>104</td>
<td>0.013</td>
</tr>
<tr>
<td>112</td>
<td>0.012</td>
</tr>
<tr>
<td>120</td>
<td>0.011</td>
</tr>
</tbody>
</table>
Example - Partially Grouted Walls

**Given:** 8 in. CMU wall; 12 ft high; Grade 60 steel, \( f_m' = 2000 \) psi; Wind load of \( w_w = 30 \) psf

**Required:** Reinforcing (place in center of wall)

**Solution:**

Determine \( M_u \)

\[
M_u = \frac{w_wh^2}{8} = \frac{(30 \text{ lb/ft}^2)(12 \text{ in./ft})(12 \text{ ft})^2}{8} = 6480 \text{ lb.in.} = 540 \text{ lb.ft} \]

Use \# __ @ ___ inches (\( A_s = ____ \text{ in.}^2/\text{ft} \))

---

Example - Partially Grouted Walls

**Problem:** 72 inch is greater than effective compression width of \( 6t = 48 \) in.

**Solution:** Use reduced value of \( b \):

\[
b = \frac{48 \text{ in.} \cdot 12 \text{ in.}}{72 \text{ in.}} = 8 \text{ in./ft}
\]

Determine \( a \)

\[
a = d - \sqrt{d^2 - \frac{2M_n}{0.8f_m'b}}
\]

\[
= 3.81 \text{ in.} - \sqrt{(3.81 \text{ in.})^2 - \frac{2 \left( \frac{0.540 \text{ ft}}{0.9} \right) \left( \frac{12 \text{ in.}}{\text{ft}} \right)}{0.8 (2.0 \text{ ksi})(\frac{8 \text{ in.}}{\text{ft}})}} = 0.1505 \text{ in.}
\]

Determine \( A_{s,reqd} \)

\[
A_{s,reqd} = \frac{0.8f_{mja}}{f_y} = \frac{0.8(2.0 \text{ ksi})(\frac{8 \text{ in.}}{\text{ft}})(0.1505 \text{ in.})}{60 \text{ ksi}} = 0.0321 \text{ in.}^2/\text{ft}
\]

0.7% increase in required area of steel

- Do not worry about it
- If worried, use \( b = 6 \text{ in./ft} \) in design, which is close enough
Partially Grouted Walls - Tolerances

Placement tolerances: (3.4.B.11)
\[ d \leq 8 \text{ in.} \quad \pm \frac{1}{2} \text{ in.} \]
\[ 8 \text{ in.} < d \leq 24 \text{ in.} \quad \pm 1 \text{ in.} \]
\[ d > 24 \text{ in.} \quad \pm 1 1/4 \text{ in.} \]

Along wall: \( \pm 2 \text{ in.} \)

8 in. CMU; \( f'_{m} = 2000 \text{ psi; Grade 60} \)

Walls: Maximum Reinforcement

- Strain gradient of \( \varepsilon_{mu} \) and \( \alpha \varepsilon_{y} \), with \( \alpha = 1.5 \) for OOP loading
- \( P_u \) determined from \( D + 0.75L + 0.525Q_E \)

\[
\rho = \frac{A_s}{bd} = \frac{0.64f'_m \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - \frac{P_u}{bd}}{f_y - \min\{ \varepsilon_{mu} - \frac{d'}{d} (\varepsilon_{mu} + \alpha \varepsilon_y), \varepsilon_y \} E_s}
\]

Fully grouted with equal tension and compression reinforcement

\[
\rho = \frac{A_s}{bd} = \frac{0.64f'_m \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - \frac{P_u}{bd}}{f_y}
\]

Fully grouted with concentrated tension reinforcement, or partially grouted with neutral axis in face shell

\[
\rho = \frac{A_s}{bd} = \frac{0.64f'_m \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) - \frac{P_u}{bd}}{f_y}
\]

Partially grouted walls with concentrated tension reinforcement and neutral axis in web

\[
\rho = \frac{0.64f'_m \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha \varepsilon_y} \right) \left( \frac{b_w}{b} \right) + 0.8f'_m f_s \left( \frac{b - b_w}{bd} \right) - \frac{P_u}{bd}}{f_y}
\]
Out-of-Plane: Moment Strength

8 in. CMU wall, partially grouted, bars in center, Grade 60 steel.

\( f'_m = 2000 \text{ psi}, \text{ unless noted; higher values to meet maximum reinforcement} \)

<table>
<thead>
<tr>
<th>Reinforcement Spacing (in.)</th>
<th>Design Moment Strength, ( \phi M_n ) (kip-ft/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#4</td>
</tr>
<tr>
<td>8</td>
<td>4.51</td>
</tr>
<tr>
<td></td>
<td>( f'_m = 2150 \text{ psi} )</td>
</tr>
<tr>
<td>16</td>
<td>2.42</td>
</tr>
<tr>
<td></td>
<td>( f'_m = 2150 \text{ psi} )</td>
</tr>
<tr>
<td>24</td>
<td>1.65</td>
</tr>
<tr>
<td>32</td>
<td>1.25</td>
</tr>
<tr>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td>48</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Masonry Beams

Lintels - Arching

(State College, PA)

Lintels - Loading

Floor Loading (neglect due to arching)

Floor Loading (include full floor loading)

4 in. (typ) Opening Span

45 deg

Lintel
Beam Depth

TMS 402 Figure CC-5.2-2

Design Tip:
\[ d \geq \frac{V_u}{1.8b \sqrt{f_m}} \]
No shear reinf.

Beam Span: Beams not built integrally with supports
TMS 402 5.2.1.1.1

Span is minimum of:
- Clear span + depth of beam
- Distance between centers of supports

8 in. bearing
4 in. to center of support

Clear Span + 4 in. + 4 in. = Clear Span + 8 in.
Beam Span: Beams built integrally with supports

TMS 402 is silent

Reasonable approximation: Clear span
- without significant negative moment reinforcement
- Lee et al (1983) showed end restraint reduced deflection from 20-45% of simply support

![Diagram of beam span](image1)

Beam Span: Beams built integrally with supports

TMS 402 5.2.1.1.2

Requires negative moment reinforcement (otherwise built integrally with supports)
- Span length distance between centers of support for determining moments
- Reasonable approximation: Design for moment at face of supports

![Diagram of moment at face of support](image2)
Beams: Lateral Support

- Lateral support (TMS 402 5.2.1.2)
  - Minimum of:
    - \(32b\)
    - \(120b^2/d\)

\[32b = 32(7.625 \text{ in.}) = 244 \text{ in.}\]
\[120b^2/d = 120(7.625 \text{ in.})^2/72 \text{ in.} = 96.9 \text{ in.}\]

Brace at every roof joist, which is 5 ft spacing

Example: Beam Depth Study

<table>
<thead>
<tr>
<th>Beam Depth</th>
<th>Flexural Reinforcement</th>
<th>Comments</th>
</tr>
</thead>
</table>
| 48 inch    | \(A_{s,reqd} = 0.305\text{in.}^2\) Use 1 - No. 5 | - Minimum reinforcing controls: Use 2 – No. 4  
- Need bracing of compression region |
| 40 inch    | \(A_{s,reqd} = 0.376\text{in.}^2\) Use 1 - No. 6 | - Need bracing of compression region |
| 32 inch    | \(A_{s,reqd} = 0.492\text{in.}^2\) Use 2 - No. 5 | - Diaphragm is close enough to provide bracing |
| 24 inch    | \(A_{s,reqd} = 0.724\text{in.}^2\) Use 2 - No. 6 or 1 – No. 8 | - No. 8 would be difficult as it would require a long development length  
- 2 – No. 6 is getting crowded |
Beams: Chapter 9 requirements

- \( P_u \leq 0.05A_nf_m' \) (TMS 402 9.3.4.2.1)
- The variation in longitudinal reinforcing bars in a beam shall not be greater than one bar size. Not more than two bar sizes shall be used in a beam. (TMS 402 9.3.4.2.1.1)
- Beams are required to be fully grouted (TMS 402 9.3.4.2.4)

Beam Construction

Bars placed in bottom U-shaped unit, or knockout bond beam unit.

[Images of beam construction and bar supports]

http://bluelinxco.com/Portals/0/Concrete%20Reinforcement%20Products.pdf

http://wirebond.com/?page_id=7214

Support on chairs

Wire to web

Bar Supports - Metal

Designed to hold reinforcing steel in position until the concrete is poured and set, all metal bar supports are manufactured to meet or exceed the recommendations of the Concrete Reinforcing Steel Institute (CRSI).

http://bluelinxco.com/Portals/0/Concrete%20Reinforcement%20Products.pdf
Beam Construction: Seismic Reinforcing

Seismic Requirements (TMS 402 7.3.2.3.1)

- \(d\) = depth to centroid of tension reinforcement
- For one layer of reinforcement, \(d \sim (d_v - 4\text{in.})\)

Example: Beam

**Given:** 10 ft. opening; superimposed dead load of 1.5 kip/ft; live load of 1.5 kip/ft;
**Required:** Design beam
**Solution:**

Material Properties:

Geometry:

Flexural Design:

Shear Design:

What else?
Example: Beam

Given: 10 ft. opening; superimposed dead load of 1.5 kip/ft; live load of 1.5 kip/ft; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; \( f'_{m} = 2000 \text{ psi} \)

Solution:
Length of bearing: Assume to be 8 in.

Span length: \( L = 10 \text{ ft} + 2(4 \text{ in.}) = 10.67 \text{ ft} \)

Lateral support of compression face:
- \( 32b = 32(7.625 \text{ in.}) = 244 \text{ in.} = 20.3 \text{ ft} \)
- \( 120b^2/d = 120(7.625 \text{ in.})^2 / (20 \text{ in.}) = 349 \text{ in.} = 29.1 \text{ ft} \)

Depth of beam:
- \( V_u \sim \frac{w_u L}{2} = \frac{1}{2} \left( 1.2 \left( 1.5 \frac{k}{\text{ft}} \right) + 1.6 \left( 1.5 \frac{k}{\text{ft}} \right) \right) (10.67 ft) = 22.4k \)
- \( d \geq \frac{V_u}{1.8b \sqrt{f'_{m}}} = \frac{22400 \text{lb}}{1.8(7.625 \text{ in.}) \sqrt{2000 \text{ psi}}} = 36.5 \text{ in.} \) Use 40 in. deep beam

Due to architectural reasons, beam depth is limited to 24 in.

---

Example: Beam, Flexure

Factored Load  
Weight of fully grouted normal weight: 83 psf

\[ w_u = 1.2D + 1.6L = 1.2 \left( 1.5 \frac{k}{\text{ft}} + 0.083 \frac{k}{\text{ft}^2} (2\text{ ft}) \right) + 1.6 \left( 1.5 \frac{k}{\text{ft}} \right) = 4.40 \frac{k}{\text{ft}} \]

Factored Moment

\[ M_u = \frac{w_u L^2}{8} = \frac{(4.40 \frac{k}{\text{ft}})(10.67 \text{ ft})^2}{8} = 62.6k \cdot \text{ft} \]

Find \( a \)
Depth of equivalent rectangular stress block

\[ a = d - \sqrt{d^2 - \frac{2M_n}{0.8f'_{m}b}} = 20 \text{ in.} - \sqrt{(20 \text{ in.})^2 - \frac{2(62.6k \cdot \text{ft})}{0.8(2.0 \text{ ksi})(7.625 \text{ in.})}} = 3.78 \text{ in.} \]

Find \( A_{s,reqd} \)
Req'd area of steel

\[ A_{s,reqd} = \frac{0.8f'_{m}ba}{f_y} = \frac{0.8(2.0 \text{ ksi})(7.625 \text{ in.})(3.78 \text{ in.})}{60 \text{ ksi}} = 0.77 \text{ in.}^2 \]

Use 2 - #6 (\( A_s = 0.88 \text{ in}^2 \))

\( M_n = 78.5 \text{ k-ft} \)
\( \phi M_n = 70.6 \text{ k-ft} \)
Example: Beam, Min and Max Reinforcement

**Minimum Reinforcement Check:** \( f_r = 160 \text{ psi} \) (parallel to bed joints in running bond; fully grouted)

- Section modulus: \( S_n = \frac{bh^2}{6} = \frac{(7.625\text{ in.})(24\text{ in.})^2}{6} = 732\text{ in.}^3 \)
- Cracking moment: \( M_{cr} = S_nf_r = 732\text{ in.}^3 (160\text{ psi}) = 117.1\text{ k in.} = 9.76\text{ k ft} \)

By inspection, \( 1.17M_{cr} \leq M_u = 62.6\text{ k ft} \)

**Maximum Reinforcement Check:** \( \rho_{max} = 0.00952 \)

\[
\rho = \frac{A_s}{bd} = \frac{0.88\text{in.}^2}{(7.625\text{in.})(20\text{in.})} = 0.00577
\]

---

Example: Beam, Development

\[
l_{de} = \frac{0.13d_b^2f_yY_g}{K\sqrt{f_m}} \quad \text{Cover} = 1.25 \text{ (face shell thickness)} + 0.5 \text{ (coarse grout)} + 0.375 \text{ (assumed #3 stirrup)} = 2.125 \text{ in.}
\]

\( K = \min\{\text{masonry cover, clear spacing between adjacent splices, } 9d_b\} \)

Determine \( K \)

\( K = \min\{2.125, 9(0.75)\} = 2.125 \text{ in.} \)

Determine \( l_{de} \)

\[
l_{de} = \frac{0.13(0.75\text{ in.})^2(60000\text{ psi})(1.3)}{2.125\text{ in.}\sqrt{2000\text{ psi}}} = 60.0\text{ in.} = 5.00\text{ ft}
\]
Example: Beam, Shear Diagram

Shear Diagram

Example: Beam, Shear

Shear at reaction

\[ V_u = \frac{w_u L}{2} = \frac{1.2 \left( \frac{k}{10000} + 0.08 \frac{k}{10000} (2 \text{ft}) + 1.6 \left( \frac{k}{10000} \right) \right) (10.67 \text{ ft})}{2} = 23.47 \text{k} \]

\( d/2 \) from face of support

\( \left( \frac{20 \text{ in.}}{2} + 4 \text{ in.} \right) \frac{1 \text{ ft}}{12 \text{ in.}} = 1.17 \text{ ft} \)

Design shear force

\[ V_u = 23.47 \text{k} \left( \frac{5.33 \text{ ft} - 1.17 \text{ ft}}{5.33 \text{ ft}} \right) = 18.33 \text{k} \]

Design masonry shear strength

\[ \phi V_{nm} = 2.25 A_{nv} \sqrt{f_m'} \]

\[ = 0.8(2.25)(7.625 \text{ in.})(20 \text{ in.}) \sqrt{2000 \text{ psi}} = 12.27 \text{k} \]

Check max \( V_n \)

\[ \phi (V_n)_{max} = 4 A_{nv} \sqrt{f_m'} \]

\[ = 0.8(4)(152.5 \text{ in.}^2) \sqrt{2000 \text{ psi}} = 21.82 \text{k} \]

\[ V_u = 18.33 \text{k} \leq 21.82 \text{k} \]

OK

Requirement for shear at \( d/2 \) from face of support is in ASD, not SD, but assume it applies
Example: Beam, Shear Reinforcement

Req’d \( V_{ns} \)

\[
V_{ns,req} = \frac{V_u - \phi V_{nm}}{\phi} = \frac{18.33 - 12.27}{0.8} = 7.58 \text{ kips}
\]

Determine \( A_v \) for a spacing of 8 in.

\[
V_n = 0.5 \left( \frac{A_v}{s} \right) f_y d_v \Rightarrow A_v = \frac{V_{ns} s}{0.5 f_y d_v}
\]

\[
A_v = \frac{7.58 \text{k}(8 \text{ in.})}{0.5(60 \text{ksi})(20 \text{ in.})} = 0.101 \text{in.}^2
\]

Use #3 stirrups

Example: Beam, Shear Details

9.3.4.2.3 Transverse reinforcement

When transverse reinforcement is required, the following provisions shall apply:

(a) Transverse reinforcement shall be a single bar with a 180° hook at each end.
(b) Transverse reinforcement shall be hooked around longitudinal reinforcement.
(c) The minimum area of transverse reinforcement shall be \( 0.0007 b d_v \).
(d) The first transverse bar shall not be located no more than one-fourth of the beam depth, \( d_v \), from the end of the beam.
(e) The maximum spacing shall not exceed 1/2 the depth of the beam nor 48 in.

Intent of provision (c) is area over a length of \( d_v \).

\[
\frac{A_v}{s} = \frac{0.11 \text{in.}^2}{8 \text{ in.}} = 0.0138 \text{in.} \geq 0.0007 b = 0.007(7.625 \text{in.}) = 0.0053 \text{in.}
\]

Development length

\[
l_{de} = \frac{0.13 d_w^2 f_y}{K \sqrt{f_m}} = \frac{0.13(0.375)^2(60000)(1.0)}{\min\{9(0.375), 1.25 + 0.5\}\sqrt{2000}} = 14.0 \text{in.}
\]

With hook:

\[
l_{de} = 14.0 \text{in.} - 13(0.375 \text{in.}) = 9.1 \text{in.}
\]
Example: Beam, Shear Details

Check width:
2(1.25 in.) Face shells
2(0.75 in.) #6 bars
2(0.5 in.) coarse grout space
2(0.375 in.) #3 stirrups
1.0 in. space between bars
Total 6.75 in. OK

Need stirrups over first
2.54 ft = 31 in.
First stirrup is at 4 in.
Use stirrups over next 32 in.
Use #3 stirrups at 8 in.

Example: Beam, Alternate Shear Details

Check width:
2(1.25 in.) Face shells
2(0.75 in.) #6 bars
2(0.5 in.) coarse grout space
2(0.375 in.) #3 stirrups
1.0 in. space between bars
Total 6.75 in. OK

Need stirrups over first
2.54 ft = 31 in.
First stirrup is at 4 in.
Use stirrups over next 32 in.
Use #3 stirrups at 8 in.
Example: Beam, Alternate Shear Details

6.1.7.2.2 Pairs of U-stirrups or ties placed to form a closed unit shall be considered properly spliced when length of laps are $1.7l_y$. In grout at least 18 in. deep, such splices with $A_vf_y$ not more than 9,000 lb per leg shall be permitted to be considered adequate if legs extend the full available depth of grout.

$$A_vf_y = (0.11\text{in.}^2)(60000\text{psi}) = 6600\text{lb}$$

Example: Beam, Deflections

Check $L/d$ ratio

$$L/d = \frac{10.67\text{ft}}{20\text{in.}} \cdot \frac{12\text{in.}}{\text{ft}} = 6.4$$

Since $L/d < 8$, deflections are OK; check to show process.

ASD load

$$w = D + L = \left(1.5 \frac{k}{ft} + 2\text{ft} \left(0.083 \frac{k}{ft^2}\right)\right) + 1.5 \frac{k}{ft} = 3.17 \frac{k}{ft}$$

Find $M_a$

$$M_a = \frac{wl^2}{8} = \frac{3.17 \frac{k}{ft} (10.67\text{ft})^2}{9} = 45.0k \cdot \text{ft}$$

Modulus ratio

$$n = \frac{E_s}{E_m} = \frac{29000\text{ksi}}{900f_m^L} = \frac{29000\text{ksi}}{900(2\text{ksi})} = 16.11$$

Reinforcement ratio

$$\rho = 0.00577 \quad np = 0.09296$$

Find $k$

$$k = \sqrt{(n\rho)^2 + 2np - n\rho}$$

$$= \sqrt{(0.0930)^2 + 2(0.0930) - 0.0930} = 0.348$$

$$kd = 0.348(20\text{in.}) = 6.96\text{in.}$$
Example: Beam, Deflections

Net moment of inertia, \( I_n \)

\[
I_n = \frac{bh^3}{12} = \frac{7.625\text{in.}(24\text{in.})^3}{12} = 8784\text{in.}^4
\]

Cracked moment of inertia, \( I_{cr} \)

\[
I_{cr} = \frac{bk^3d^3}{3} + nA_s(d - kd)^2
= \frac{7.625\text{in.}(6.96\text{in.})^3}{3} + 16.11(0.88\text{in.}^2)(20\text{in.} - 6.96\text{in.})^2
= 3268\text{in.}^4
\]

Effective moment of inertia, \( I_{eff} \)

\[
I_{eff} = I_n \left( \frac{M_{cr}}{M_a} \right)^3 + I_{cr} \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right]
= 8784\text{in.}^4 \left( \frac{9.76\text{k-ft}}{45.0\text{k-ft}} \right) + 3268\text{in.}^4 \left[ 1 - \left( \frac{9.76\text{k-ft}}{45.0\text{k-ft}} \right)^3 \right] = 3324\text{in.}^4
\]

Deflection, \( \delta \)

\[
\delta = \frac{5wl^4}{384EI} = \frac{5(3166\text{lb})(10.67\text{ft})^4}{384(1800000\text{psi})(3324\text{in.}^4)} \frac{1728\text{in.}^3}{1\text{ft}^3} = 0.154\text{in.}
\]

Allowable \( \delta \)

\[
\frac{L}{600} = \frac{10.67\text{ft}}{600} \frac{12\text{in.}}{ft} = 0.213\text{in.} \quad \text{OK}
\]

Deep Beams, 5.2.2

- Lintels in which there is a large height of masonry above the opening
- Walls not continuously supported (supported on pier foundations)

- Plane sections do not remain plane
- Internal level arm smaller than computed from linear strain distribution

Effective span length, \( L_{eff} \), smaller of:

- center-to-center distance between supports
- \( 1.15 \) multiplied by the clear span

\[
\frac{l_{eff}}{d_v} \leq \begin{cases} 
3 & \text{continuous span} \\
2 & \text{simple span}
\end{cases}
\]

\[
z = \begin{cases} 
0.2(l_{eff} + 2d_v) & 1 \leq \frac{l_{eff}}{d_v} < 2 \\
0.6l_{eff} & \frac{l_{eff}}{d_v} < 1
\end{cases}
\]

\[
\frac{l_{eff}}{d_v} \leq \begin{cases} 
3 & \text{continuous span} \\
2 & \text{simple span}
\end{cases}
\]

\[
z = \begin{cases} 
0.2\left( l_{eff} + 1.5d_v \right) & 1 \leq \frac{l_{eff}}{d_v} < 3 \\
0.5l_{eff} & \frac{l_{eff}}{d_v} < 1
\end{cases}
\]

Reinforced Masonry - Flexural Members 62
Deep Beams

- Flexural reinforcement
  - distributed flexural reinforcement for half beam depth
  - maximum spacing of one-fifth \( d_v \) or 16 in.
  - joint reinforcement can be used as flexural reinforcement
  - horizontal reinforcement anchored to develop yield strength at face of supports

- Shear reinforcement (when required)
  - minimum area of vertical reinforcement is \( 0.0007bd_v \)
  - horizontal shear reinforcement shall have area \( \geq \) half vertical shear reinforcement
  - maximum spacing of shear reinforcement shall be one-fifth \( d_v \) or 16 in.

- Total reinforcement: sum of horizontal and vertical reinforcement shall be at least \( 0.001bd_v \).

Deep Beams – Example 1

Given: 10 ft. opening; 6 ft. deep beam; dead load (including beam) of 3.0 kip/ft; live load of 2.0 kip/ft; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; \( f_m' = 2000 \text{ psi} \)

Required: Design beam

Solution:

Center-to-center between supports = 10 ft + 2(4 in.) = 10.67 ft
1.15(clear span) = 1.15(10 ft) = 11.5 ft
Effective span length, \( l_{eff} = \min(10.67, 11.5) = 10.67 \text{ ft} \)

Factored moment, \( M_u \)
\[
M_u = \frac{1.2 \left(3.0 \frac{k}{ft} \right) + 1.6 \left(2.0 \frac{k}{ft} \right) \times (10.67 \text{ft})^2}{8} = 96.7 k \cdot ft
\]

Span ratio, \( l_{eff} / d_v \)
\[
\frac{l_{eff}}{d_v} = \frac{10.67 \text{ft}}{6 \text{ft}} = 1.78
\]

Internal lever arm, \( z \)
\[
z = 0.2 \left(l_{eff} + 2d_v\right) = 0.2 \left(10.67 \text{ft} + 2(6 \text{ft})\right) = 4.53 \text{ft} \]
Deep Beams – Example 1, Flexure

Req'd \( A_s \)

\[
A_{s,reqd} = \frac{M_u/\phi}{zf_y} = \frac{96.7k \cdot ft/0.9}{4.53ft(60ksi)} = 0.395\text{in.}^2
\]

Using standard beam theory, \( A_{s,reqd} = 0.320 \text{ in}^2 \) (19% less)

**Use 2-#4 bars**

Flexural reinforcement requirements:
- distributed flexural reinforcement for half beam depth
  - need flexural reinforcement over bottom 36 inches.
- maximum spacing of one-fifth \( d_v \) or 16 in.
  - \((1/5)(72) = 14.4 \text{ in.}\)
- Use W1.7 (9 gage) joint reinforcement every 8 in. in bottom 5 bed joints (as a practical matter, use in every bed joint in beam)

Deep Beams – Example 1, Shear

Factored shear, \( V_u \)

\[
V_u = \frac{1.2(3.0\frac{k}{ft})+1.6(2.0\frac{k}{ft})(10.67\text{ft})}{2} = 36.3\text{kip}
\]

Design shear strength, \( \phi V_{nm} \)

\[
\phi V_{nm} = \phi 2.25A_{nv}\sqrt{f_m} = 0.8(2.25)(7.625\text{in.})(68\text{in.})\sqrt{2000}\text{psi} = 41.7\text{k}
\]

No shear reinforcement required

Total reinforcement

\[
0.001bd_v = 0.001(7.625\text{in.})(72\text{in.}) = 0.55\text{ in}^2
\]

2-#4 (0.40 in.\(^2\)) + 5(2)(0.017 in.\(^2\)) = 0.57 in.\(^2\)

OK
Deep Beams – Example 1, Development

Clear cover
1.25 (face shell) + 0.5 (coarse grout) = 1.75 in.

Development length, $l_{de}$

\[
l_{de} = \frac{0.13d^2_{p}f_{y}g}{K\sqrt{f'_m}} = \frac{0.13(0.5in.)^2(60000psi)(1.0)}{\min\{9(0.5in), 1.75in.\} \sqrt{2000psi}} = 24.9\text{in.}
\]

Extend bars 24 in. beyond face of support
(close enough; cover of 1.82 in., 1/16 in. more than 1.75 in., results in $l_{de} = 24$ in.)

Deep Beams – Example 2

Given: 10 ft. opening; 6 ft. deep beam; dead load (including beam) of 6.0 kip/ft; live load of 3.0 kip/ft; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; $f'_m = 2000$ psi

Required: Design beam

Solution:

Factored moment, $M_u$

\[
M_u = \frac{1.2(6.0\frac{k}{ft})+1.6(3.0\frac{k}{ft})}{8}(10.67ft)^2 = 170.7\text{k} \cdot \text{ft}
\]

Req’d $A_s$

\[
A{s,reqd} = \frac{M_u/\phi}{z_{fy}} = \frac{170.7k \cdot \text{ft}/0.9}{4.53\text{ft}(60ksi)} = 0.697\text{in.}^2
\]

Use 2-#4 bars in each of bottom 2 courses (4 total bars)

Use joint reinforcement in every course for distributed reinforcement

Using standard beam theory, $A{s,reqd} = 0.607\text{ in}^2$ (13% less)
Deep Beams – Example 2, Shear

Factored shear, $V_u$

$$V_u = \frac{\left[12\left(6.0\frac{k}{ft}\right)+1.6\left(3.0\frac{k}{ft}\right)\right](10.67ft)}{2} = 64.0kip$$

Max shear

$$\phi V_{n,max} = 0.8\left[4A_{nv}\sqrt{f_m}\right] = 0.8\left[4(518in.)^2\sqrt{2000psi}\right] = 74.1kip$$

Req’d shear strength, $V_{ns}$

$$\phi V_{ns,reqa} = 64.0k - 41.7k = 22.3k$$

Req’d shear steel, $A_v$

$$A_v = \frac{V_{ns}^s}{0.5f_yd_y} = \frac{(22.3k/0.8)(8in.)}{0.5(60ksi)(72in.)} = 0.103in.^2$$

Use 1-#3 at 8 in. vertical shear reinforcement

Deep Beams – Example 2, Shear Details

- Minimum area of vertical reinforcement is $0.0007bd_v$
  - $0.0007bd_v = 0.0007(7.625)(72) = 0.384 in.^2$
  - Total vertical is $0.11in.^2(15) = 1.65 in.^2$

- Horizontal shear reinforcement shall have area $\geq$ half vertical shear reinforcement
  - Use 1-#4 each course
  - Do not need joint reinforcement

- Maximum spacing of shear reinforcement shall be one-fifth $d_v$ or 16 in.
  - Maximum spacing is 14.4 in.